ON RADIAL SPRING CONSTANTS AT THE JUNCTURE OF A RADIAL NOZZLE AND A SPHERICAL SHELL

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ABSTRACT

This paper presents the values of the radial spring constant at the juncture of a radial nozzle and a spherical shell due to radial load. The spring constant is a function of diameters and thicknesses of the nozzle and the vessel. It is known that a lower spring constant at the nozzle-shell juncture would reduce the local peak stresses which is very important in fatigue design of the pressure vessel. This study shows that the value of the radial spring constant may be reduced by decreasing the thicknesses of both shell and nozzle, decreasing nozzle diameter and increasing the shell diameter. Analytical derivations for this spring constant are presented in this paper. The actual spring constants for various nozzle-shell combinations have been computed using a digital computer. These values are tabulated and plotted in this paper to facilitate the vessel design and stress analysis.

NOMENCLATURE

R = Radius of middle plane of spherical shell

t = Thickness of the spherical shell

a = Radius of the middle surface of the radial nozzle

h = Thickness of the radial nozzle

l = $[R^2 t^2/(12\,(1-\mu^2))]^{1/4}$

r = Radius of spherical shell section in a latitudinal plane (Refer to Fig. 1)

s = $r/l = 1.81784\,(r/R)\cdot(R/t)^{1/2}$

u = $a/l = 1.81784\,(a/R)\cdot(R/t)^{1/2}$

T = Temperature of operation

v = Radial deflection of nozzle; positive if away from axis of the nozzle

w = Radial deflection of spherical shell, positive if away from centre of shell

F = Stress function

K_R = Spring constant for the case of radial load

x = Axial coordinate of cylindrical shell (nozzle)

y = Tangential coordinate for cylindrical and spherical shells

z = Radial coordinate of cylindrical shell

β = R/t (A Shell Parameter)

γ = a/h (A Nozzle Parameter)

ρ = t/h (Thickness Ratio)

β_o = $[3(1-\mu^2)/(a^2h^2)]^{1/4}$

η = $(u^4\rho^2/\gamma^2)/[24(1-\mu^2)]$

μ = Poisson's Ratio (Assumed as 0.3 for calculations)

E_c = Modulus of elasticity at room temperature

E_h = Modulus of elasticity at temperature of operation

A_1 to A_4 = Constants

C_3, C_4, C_{12} = Constants

D = $Et^3/[12(1-\mu^2)]$ = Flexural Rigidity of spherical shell

N = $Eh^3/[12(1-\mu^2)]$ = Flexural Rigidity of cylindrical shell (nozzle)

N_x = Radial membrane force, acting per unit width upon a normal section of the spherical shell

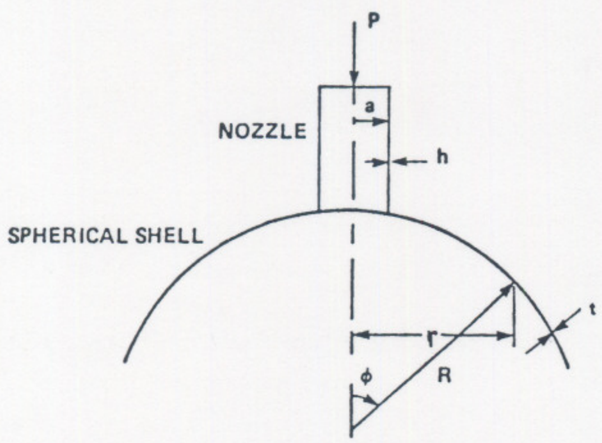

FIG. 1 SPHERICAL SHELL SUBJECTED TO A RADIAL LOAD ACTING UPON A NOZZLE

N_y = Tangential membrane force, acting per unit width upon a meridional section of the spherical shell

M_o = Bending moment M_x in cylindrical shell (nozzle) at $x = o$

V_o = Transverse shear V_x in cylindrical shell at $x = o$

P = Radial Load (coming through a nozzle)

$\propto$ = Slope angle at juncture of spherical and cylindrical shell with respect to their original positions due to their deflections

ϵ = Unit Strain

ϕ = Angle between normal to spherical shell-middle surface and shell axis, in radians

θ = Polar coordinate for cylindrical and spherical shells, in radians

Ker s, Kei s, Ker u, Kei u = Kelvin functions of zero order

Ker $'s$, Kei $'s$, Ker $'u$, Kei $'u$ = Derivatives of Kelvin functions

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r} \cdot \frac{d}{dr}$$

INTRODUCTION

With the advent of more and more chemical plants, power plants and other process plants, the safety requirement and economy in design of pressure vessels are gaining high priority. This leads to the requirement of a more accurate stress analysis.

It is known that there exist highly localized stresses at the juncture of nozzle and pressure vessels, both in cylindrical and spherical types. However these stresses can not be accurately evaluated even with most modern and highly sophisticated computer programs without reliable values of stiffness or spring constants at these junctures. The spring constants at the juncture of a nozzle and a cylindrical shell has been studied by Murad [1]. This paper deals specifically with radial spring constants at the juncture of a nozzle and a spherical shell. The rotational spring constants will be presented in a separate paper.

Bijlaard has studied the differential equations for a radial load acting on a spherical shell. His solution leads to the deflection of the spherical shell only [2]. He did not specify the relationships of spring constants in terms of shell parameters (diameter and thickness) and the nozzle parameters (diameter and thickness). This part is explicitly developed here. The solution involves Kelvin function of zero and first orders along with their derivatives. These are obtained by programming the mathematical equations given by Abramowitz and Stegun [3].

DERIVATION OF EXPRESSION FOR DEFLECTION OF SPHERICAL SHELL DUE TO A RADIAL LOAD

The two governing simultaneous equations for shallow shells with distributed normal load P are given by [4]:

$$\nabla^4 F - (tE/R)\ \nabla^2 w = 0 \tag{1}$$

$$\nabla^4\ w + (1/RD)\ \nabla^2 F = P/D \tag{2}$$

where w = Radial deflection in the direction of exterior normal.

F = Stress function.

D = Flexural rigidity of the spherical shell.

The solution of above two simultaneous differential equations is given by Ref. [2]:

$$w = C_3 \text{ Ker } s + C_4 \text{ Kei } s \tag{3}$$

$$F = \left[\frac{Et^3}{12\,(1-\mu^2)^{1/2}}\right] \cdot (C_3 \text{ Kei } s - C_4 \text{ Ker } s + C_{12}\ \ln s) \tag{4}$$

where $C_{12} = -\,[3(1-\mu^2)]^{1/2} \cdot P \cdot R/(\pi E t^2)$ (5)

$$s = (r/1) = 1.81784\,(r/R) \cdot (R/t)^{1/2} \tag{6}$$

$$1^4 = R^2 t^2/[12(1-\mu^2)] \tag{7}$$

The constants C_3 and C_4 have to be determined from the two boundary conditions at $r = a$ as follows:

BOUNDARY CONDITION (I)

(1) At the juncture of the shell and nozzle, the angle

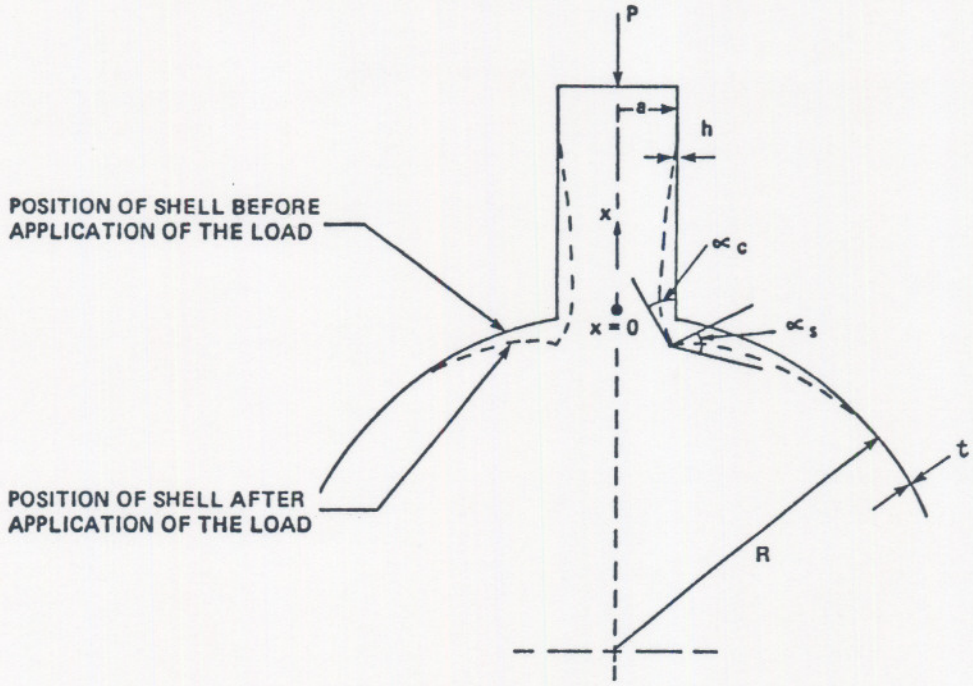

FIG. 2 POSITION OF SHELL BEFORE AND AFTER APPLICATION OF RADIAL LOAD

of rotation for the spherical shell and the nozzle (considered as a cylindrical shell) should be equal (Refer to Fig. 2), i.e.

$$(\alpha_s)_{r=a} = (\alpha_c)_{x=0} \tag{8}$$

BOUNDARY CONDITION (II)

(2) The strain ϵ_y in the tangential direction should be equal for spherical and cylindrical shell, i.e.,

$$(\epsilon_{ys})_{r=a} = (\epsilon_{yc})_{x=0} \tag{9}$$

Boundary Condition (I) is obtained from Eqs. (6) and (8) of Ref. [2]. These equations are numbered in this paper as (A) and (B) respectively and are given as follows:

$$(\alpha_s)_{r=a} = (dw/dr)_{r=a} = (1/l)(C_3 \text{ Ker}'u + C_4 \text{ Kei}'u) \tag{A}$$

$$(\alpha_s)_{x=0} = (-dv/d/x)_{x=0} = [1/(2\beta o^2 N)]\ (2\beta oMo + Vo) \tag{B}$$

Boundary Condition (II) is obtained from Eqs. (7) and (12) of Ref. [2]. These equations are numbered in this paper as (C) and (D) respectively and are given as follows:

$$(\epsilon_{ys})_{r=a} = (N_y - \mu Nx)/(Et) = (1/R)\ [C_3(\text{Ker } u - u^{-1} \text{ Kei}'u) + C_4 (\text{Kei } u + u^{-1} \text{ Ker}'u) - C_{12} u^{-2} - \mu(C_3 u^{-1} \text{ Kei}'u - C_4 u^{-1} \text{ Ker}'u + C_{12} u^{-2})] \tag{C}$$

$$(\epsilon_{yc})_{x=0} = [1/(2\beta o^3 Na)]\ (\beta oMo + Vo) + P/(2\pi ahE) \tag{D}$$

where M_o and V_o are given by Eqs. (13) and (22) of Ref. [2]. These equations are numbered in this paper as (E) and (F) respectively and are given as follows:

$$M_o = (M)_{r=a} = -\frac{Et^2}{R[12(1-\mu^2)]^{1/2}} [C_3 \{\text{Kei } u + (1-\mu)u^{-1} \text{ Ker}'u\} - C_4 \{\text{Ker } u - (1-\mu)u^{-1} \text{ Kei}'u\}] \tag{E}$$

$$V_o = [ET/Ru]\ [(1+\eta)(C_3 \text{ Kei}'u - C_4 \text{ Ker}'u) + (1-\eta) C_{12} u^{-1}] \tag{F}$$

One notes that β in Eqs. (8) and (12) of Ref. [2] has been changed to β_o in Eqs. (B) and (D) above.

Upon substitution of Eqs. (A) and (B), the boundary condition (I) becomes [4]:

$$(2\beta_o{}^3 N/l)(C_3 \text{ Ker}'u + C_4 \text{ Kei}'u) = [-(2\beta_o Et^2/R)/(12(1-\mu^2))^{1/2}] \cdot [C_3 (\text{Kei } u + (1-\mu)\cdot u^{-1}\cdot \text{Ker}'u) - C_4(\text{Ker } u - (1-\mu)\cdot u^{-1}\cdot \text{Kei}'u)] + [Et/(Ru)]\cdot[(1+\eta)\cdot(C_3 \text{ Kei}'u - C_4 \text{ Ker}'u) + (1-\eta)(C_{12}\cdot u^{-1})] \quad (10)$$

Upon substitution of Eqs. (C) and (D), the boundary condition (II) becomes [4]:

$$(2\beta_o{}^3 Na/R)[C_3(\text{Ker } u - u^{-1} \text{ kei}'u) + C_4(\text{Kei } u + u^{-1}\cdot \text{Ker}'u) - C_{12}\cdot u^{-2} - \mu(C_3\cdot u^{-1}\cdot \text{Kei}'u - C_4\cdot u^{-1}\cdot \text{Ker}'u + C_{12}\cdot u^{-2})] = -[(\beta_o Et^2/R)/(12(1-\mu^2))^{1/2}]\cdot[C_3(\text{Kei } u + (1-\mu)\cdot u^{-1}\cdot \text{Ker}'u) - C_4(\text{Ker } u - (1-\mu)\cdot u^{-1}\cdot \text{Kei}'u)] + [Et/(Ru)]\cdot[(1+\eta)\cdot(C_3 \text{ Kei}'u - C_4\cdot \text{Ker}'u) + (1-\eta)\cdot C_{12}\cdot u^{-1}] + \mu\cdot\beta_o{}^3\cdot NP/(\pi hE) \quad (11)$$

Solving Eqs. (10) and (11) for C_3 and C_4 we get:

$$C_3 = \left[\frac{A_4}{A_2}\frac{6(1-\mu^2)\cdot(1-\eta)\gamma\rho}{\pi(12(1-\mu^2))^{1/4}u^2} - \frac{(12(1-\mu^2))^{1/2}}{2\pi u^2}(1+\mu+[48(1-\mu^2)]^{1/4}\cdot(1-\eta)\cdot\rho\cdot\gamma^{1/2}) + \frac{\mu\cdot\rho^2}{2\pi\cdot\gamma}\right]\cdot\frac{A_2}{(A_2A_3 - A_1A_4)}\cdot\frac{PR}{Et^2} \quad (12)$$

$$C_4 = -\frac{1}{A_2}\left[A_1C_3 + \frac{6(1-\mu^2)}{\pi(12(1-\mu^2))^{1/4}}\cdot(1-\eta)\cdot\frac{\gamma\rho}{u^2}\cdot\frac{PR}{Et^2}\right] \quad (13)$$

where A_1, A_2, A_3, and A_4 are given as follows:

$$A_1 = [12(1-\mu^2)]^{1/4}\cdot[(\gamma/\rho)\text{ Ker}'u - \gamma\rho(1+\eta)\cdot\text{Kei}'u]\cdot u^{-1} + \rho^2(2\gamma)^{1/2}\cdot[\text{Kei } u + (1-\mu)\cdot u^{-1}\cdot\text{Ker}'u] \quad (14)$$

$$A_2 = [12(1-\mu^2)]^{1/4}\cdot u^{-1}[(\gamma/\rho)\cdot\text{Kei}'u + \gamma\rho(1+\eta)\cdot\text{Ker}'u] - \rho^2\cdot(2\gamma)^{1/2}[\text{Ker } u - (1-\mu)\cdot u^{-1}\cdot\text{Kei}'u] \quad (15)$$

$$A_3 = \text{Ker } u - (1+\mu)\cdot u^{-1}\cdot\text{Kei}'u + \rho^2[\text{Kei } u + (1-\mu)\cdot u^{-1}\cdot\text{Ker}'u] - [48(1-\mu^2)]^{1/4}\cdot\rho\gamma^{1/2}\cdot u^{-1}\cdot(1+\eta)\cdot\text{Kei}'u \quad (16)$$

$$A_4 = \text{Kei } u + (1+\mu)\cdot u^{-1}\cdot\text{Ker}'u - \rho^2[\text{Ker } u - (1-\mu)\cdot u^{-1}\cdot\text{Kei}'u] + [48(1-\mu^2)]^{1/4}\cdot\rho\cdot\gamma^{1/2}\cdot u^{-1}(1+\eta)\cdot\text{Ker}'u \quad (17)$$

Thus the final solution for deflection of spherical shell due to a radial load P coming through the nozzle is given by Eq. (3) with constants C_3 and C_4 given by Eqs. (12) and (13) and constants A_1 to A_4 given by Eqs. (14) to (17).

APPROACH FOR EVALUATION OF RADIAL SPRING CONSTANT

The expression for deflection (w) due to a radial load is derived above and is given by:

$$w = C_3 \text{ Ker } s + C_4 \text{ Kei } s \quad (3)$$

where C_3 and C_4 are given by Eqs. (12) and (13) and constants A_1 to A_4 are given by Eqs. (14) to (17) respectively.

Since we are interested only in the deflection of the vessel at the juncture of nozzle, we let $r = a$.

At $r = a$, $s = (a/l) = u$. Hence the expression for w in this case becomes:

$$w = C_3 \text{ Ker } u + C_4 \text{ Kei } u$$

where $$u = 1.81784(a/R)\sqrt{(R/t)} \quad (18)$$

Given a deflection (w) corresponding to a load P, the spring constant can be written as:

$$K_R = P/|w| \quad (19)$$

Taking P = 1.0 lb, the expression for spring constant becomes:

$$K_R = 1.0/[C_3 \text{ Ker } u + C_4 \text{ Kei } u] \quad (20)$$

For a given set of shell parameters (R and t) and nozzle parameters (a and h), unique values of u, C_3 and C_4 exist. Hence given R, t, a and h, unique values of deflection w and spring constants K_R can be obtained.

For convenience, we define three independent dimensionless parameters β, γ and ρ as follows:

Shell parameter : $\beta = R/t$

Nozzle parameter: $\gamma = a/h$

Shell to nozzle thickness ratio: $\rho = t/h$

Fixing the values of β, γ and ρ does not fix R, t, a and h, e.g., if R, t, a and h are doubled, the ratios, β, γ and ρ still remain the same while the spring constant (K_R) for this new set of values gets changed. Hence to arrive at a unique value of K_R, we must fix one more parameter apart from β, γ and ρ. This is achieved as follows: –

For given values of R, t, a and h, we find the values of u. Since we are interested in the juncture of the spherical shell and nozzle, using $u = s$ curve (corresponding to the calculated values of β, γ and ρ for above values of R, t, a and h) from a plot of $w/(RP/Et^2)$ vs s in Ref. [2], we obtain

$$\frac{wEt^2}{RP} = C_1' \text{ (Constant)}$$

Hence, spring constant, $K_R = \frac{P}{|w|} = \frac{Et^2}{C_1 R}$

where $C_1 = |C_1'| =$ Constant

$$\text{or } K_R = \frac{E}{C_1} \cdot \frac{t}{R} \cdot t \qquad (21)$$

where E is the modulus of elasticity

Eq. (21) can also be written as:

$$\frac{K_R}{t} = \frac{C_2}{\beta}$$

where $\beta = R/t$ (As per definition)

$C_2 = (E/C_1) =$ Another Constant

Hence, to fix the value of the K_R for fixed values of β, γ and ρ, we must fix t also.

At this stage, it seems logical to plot the values of K_R/t vs β for various combinations of γ and ρ because there exists a *Unique* value of K_R/t for a given set of β, γ and ρ. In fact, in a later part of this paper, values for K_R/t have been plotted against the shell parameter β (R/t) for three different values of the nozzle parameter γ (a/h) and unique values of the constant ρ (t/h).

Using all the above equations, a computer program has been written to evaluate spring constants (K_R) and ratios K_R/t for various combinations of R, t, a and h (i.e., β, γ and ρ). The various equations for Kelvin functions (Ker u, Kei u) and their derivatives (Ker $'u$, Kei $'u$) are taken from Ref. [3].

Computer results for spring constants K_R and ratio K_R/t are obtained for various values of β (Beta), γ(Gamma) and ρ(Rho). Tables 1 -4 give results for $\rho = 1.0, 2.0, 4.0, 10.0$; $\gamma = 5, 10, 15$ and various ranges of β.

The values of deflection (w) computed in this study have been verified with Bijlaard's work in Ref. [2] and are found to be matching quite closely. Values taken from Tables 1 to 4 are also plotted in this paper to facilitate the vessel design and stress analysis. These are given in Figs. 3 to 6 respectively.

One must note that the values of spring constants are valid only if the deflections are limited to a segment of shell that can be considered shallow. This leads to conditions:

(a) $u \leqslant 1.0, R/t \geqslant 10$

(b) $u > 1.0, R/t \geqslant (u + 2.3)^2$

Also, the values of spring constants are considered accurate only if a/R is less than or equal to 1/3. This leads to the following condition:

(c) $a/R \leqslant 1/3$, i.e., $u \leqslant 1.05\sqrt{\gamma/\rho}$

The above three conditions are derived in Ref. [4] and can also be found in the Appendix. The computer program is designed to take care of these three conditions.

CORRECTION OF THE SPRING CONSTANTS FOR HOT MODULUS

As we know that the value of E varies depending upon the temperature of operation. The graphs plotted here are for the cold condition, i.e., $E = 30 \times 10^6$ psi. In case, the temperature of operation is higher than the normal ambient temperature, the value of spring constant (K_R) obtained at ambient temperature must be corrected as follows:

As given by Eq. (21), we know:

$$K_R = \frac{E}{C_1} \cdot \frac{t}{R} \cdot t$$

Hence, for a given geometry of spherical shell:

$$K_R \propto E$$

$$\text{or} \quad \frac{K_R \text{ (at temperature of operation)}}{K_R \text{ (at ambient temperature)}} = \frac{E_h}{E_c}$$

$$\text{or} \quad K_R \text{ (at temperature of operation)} = \frac{E_h}{E_c} \cdot K_R \text{ (at ambient temperature)} \qquad (22)$$

$E_c = 30 \times 10^6$ psi

$E_h =$ Modulus of Elasticity at temperature of operation

Thus if β, γ, ρ, t and temperature of operation are specified, the spring constant can readily be found using these curves

Given below is an example to illustrate the use of these curves:

Numerical Example

Given: $R = 1600$ in., $t = 8$ in., $a = 30$ in., $h = 2$ in.
Temperature of Operation = 600°F
Material: Carbon Steel with Carbon content $\leqslant 0.3\%$

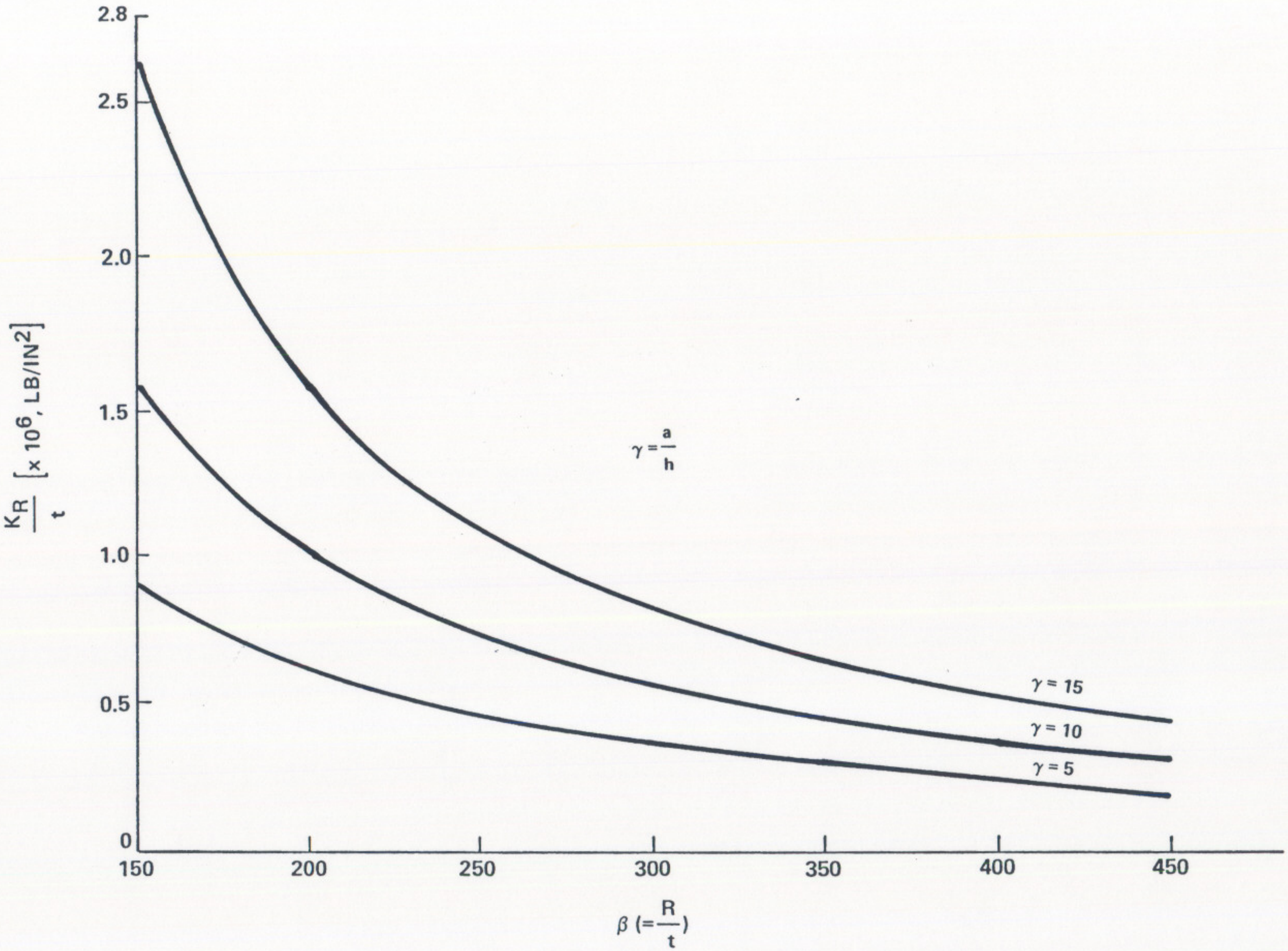

FIG. 3 GRAPH FOR RADIAL SPRING CONSTANTS FOR $\rho = 1.0$

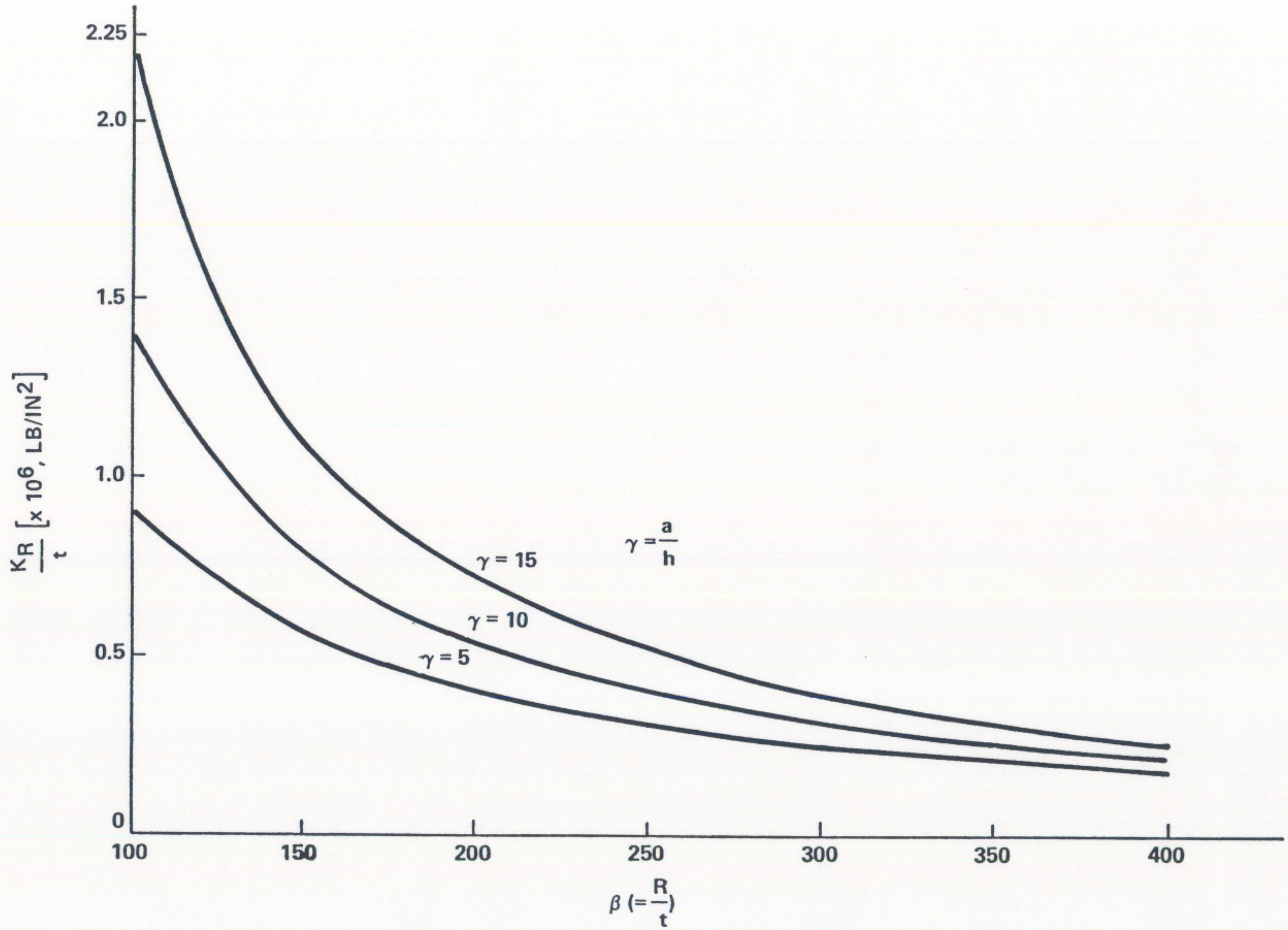

FIG. 4 GRAPH FOR RADIAL SPRING CONSTANTS FOR $\rho = 2.0$

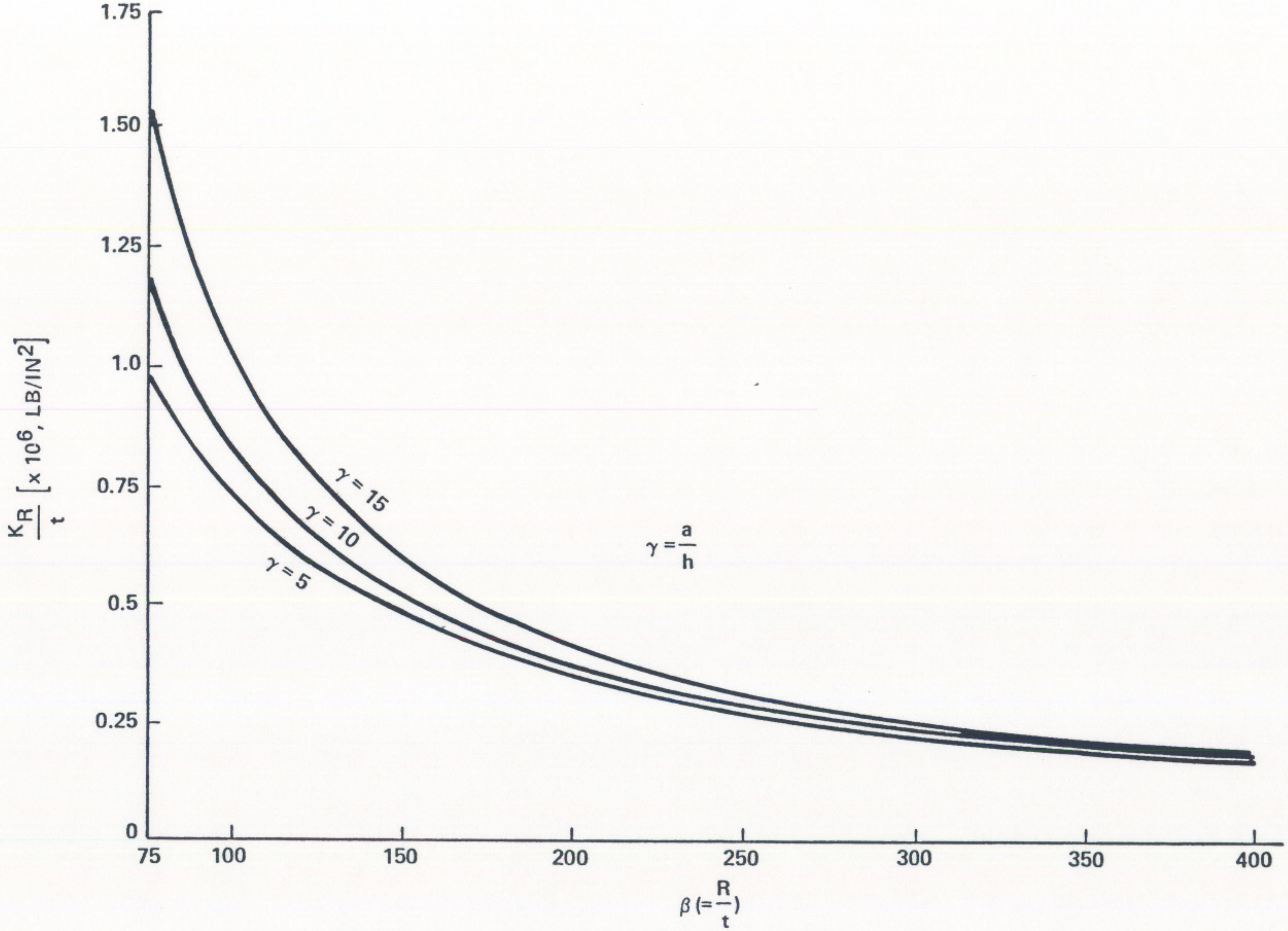

FIG. 5 GRAPH FOR RADIAL SPRING CONSTANTS FOR ρ = 4.0

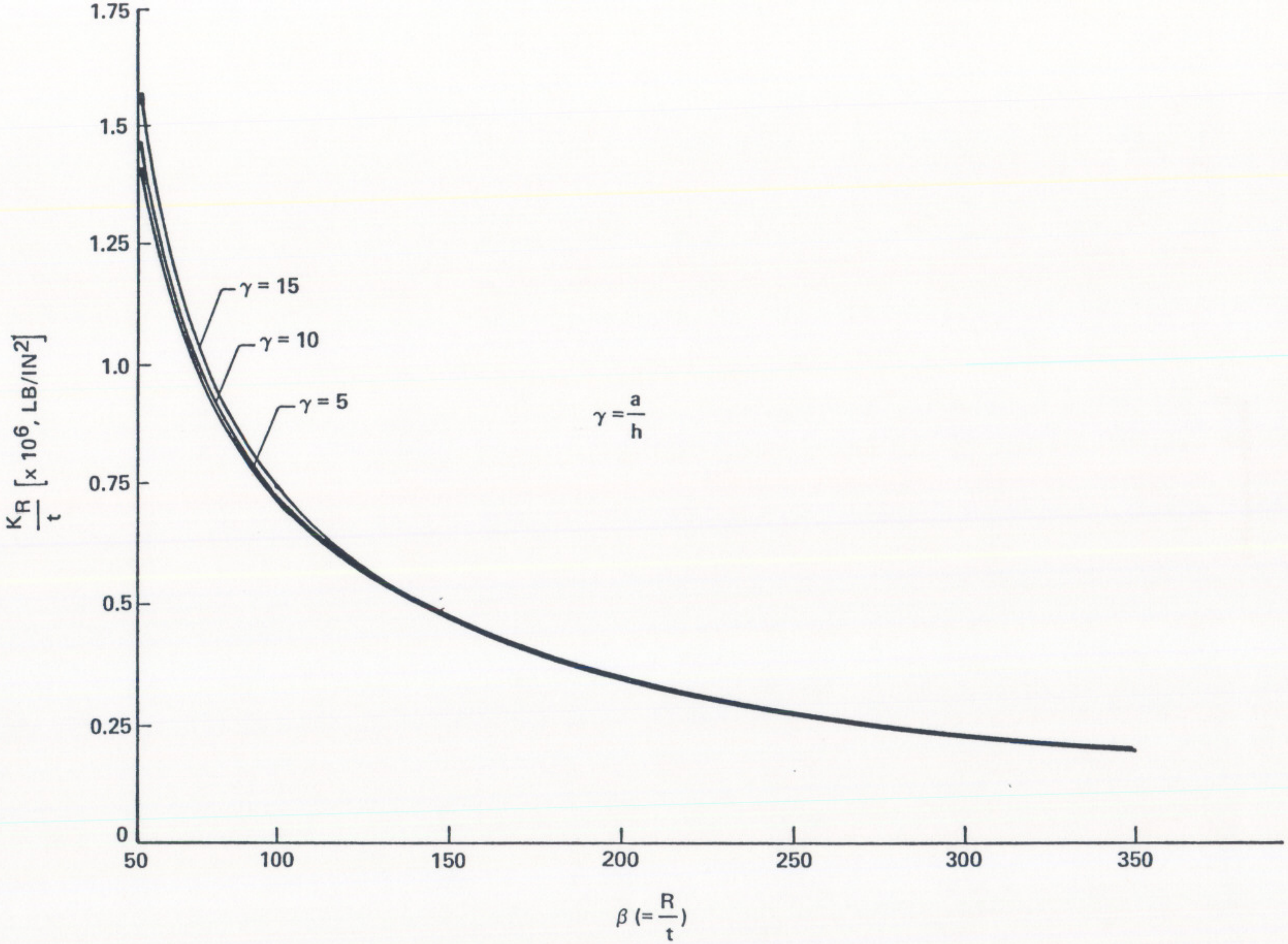

FIG. 6 GRAPH FOR RADIAL SPRING CONSTANTS FOR ρ = 10.0

TABLE 1 RESULTS FOR RADIAL SPRING CONSTANTS FOR $\rho = 1.0$

K=SPRING CONSTANT   BETA=R/T   GAMMA=A/H   RHO=T/H   U=1.81784*A*SQRT(BETA)/R

R(IN.)	T(IN.)	A(IN.)	H(IN.)	DEFLECTION(IN.)	K(LBS./IN.)	K/T	BETA	GAMMA	RHO	U
150.0000	1.0000	5.0000	1.0000	-0.000001127	0.88714E 06	0.88714E 06	150.000	5.00	1.00	0.742130
320.0000	1.6000	8.0000	1.6000	-0.000001026	0.97473E 06	0.60921E 06	200.000	5.00	1.00	0.642703
500.0000	2.0000	10.0000	2.0000	-0.000001090	0.91753E 06	0.45876E 06	250.000	5.00	1.00	0.574851
720.0000	2.4000	12.0000	2.4000	-0.000001140	0.87735E 06	0.36556E 06	300.000	5.00	1.00	0.524764
1050.0000	3.0000	15.0000	3.0000	-0.000001102	0.90781E 06	0.30260E 06	350.000	5.00	1.00	0.485838
1440.0000	3.6000	18.0000	3.6000	-0.000001079	0.92679E 06	0.25744E 06	400.000	5.00	1.00	0.454459
1800.0000	4.0000	20.0000	4.0000	-0.000001118	0.89429E 06	0.22357E 06	450.000	5.00	1.00	0.428469
2200.0000	4.4000	22.0000	4.4000	-0.000001152	0.86810E 06	0.19729E 06	500.000	5.00	1.00	0.406481
150.0000	1.0000	10.0000	1.0000	-0.000000634	0.15775E 07	0.15775E 07	150.000	10.00	1.00	1.484259
320.0000	1.6000	16.0000	1.6000	-0.000000617	0.16203E 07	0.10127E 07	200.000	10.00	1.00	1.285406
500.0000	2.0000	20.0000	2.0000	-0.000000686	0.14567E 07	0.72835E 06	250.000	10.00	1.00	1.149702
720.0000	2.4000	24.0000	2.4000	-0.000000743	0.13459E 07	0.56078E 06	300.000	10.00	1.00	1.049528
1050.0000	3.0000	30.0000	3.0000	-0.000000738	0.13553E 07	0.45176E 06	350.000	10.00	1.00	0.971676
1440.0000	3.6000	36.0000	3.6000	-0.000000739	0.13531E 07	0.37586E 06	400.000	10.00	1.00	0.908919
1800.0000	4.0000	40.0000	4.0000	-0.000000780	0.12814E 07	0.32034E 06	450.000	10.00	1.00	0.856937
2200.0000	4.4000	44.0000	4.4000	-0.000000817	0.12239E 07	0.27816E 06	500.000	10.00	1.00	0.812962
150.0000	1.0000	15.0000	1.0000	-0.000000381	0.26253E 07	0.26253E 07	150.000	15.00	1.00	2.226388
320.0000	1.6000	24.0000	1.6000	-0.000000398	0.25138E 07	0.15711E 07	200.000	15.00	1.00	1.928108
500.0000	2.0000	30.0000	2.0000	-0.000000462	0.21653E 07	0.10827E 07	250.000	15.00	1.00	1.724553
720.0000	2.4000	36.0000	2.4000	-0.000000515	0.19415E 07	0.80898E 06	300.000	15.00	1.00	1.574293
1050.0000	3.0000	45.0000	3.0000	-0.000000523	0.19114E 07	0.63713E 06	350.000	15.00	1.00	1.457513
1440.0000	3.6000	54.0000	3.6000	-0.000000534	0.18740E 07	0.52055E 06	400.000	15.00	1.00	1.363378
1800.0000	4.0000	60.0000	4.0000	-0.000000572	0.17483E 07	0.43707E 06	450.000	15.00	1.00	1.285405
2200.0000	4.4000	66.0000	4.4000	-0.000000606	0.16488E 07	0.37474E 06	500.000	15.00	1.00	1.219442

TABLE 2 RESULTS FOR RADIAL SPRING CONSTANTS FOR ρ = 2.0

K=SPRING CONSTANT   BETA=R/T   GAMMA=A/H   RHO=T/H   U=1.81784*A*SQRT(BETA)/R

R(IN.)	T(IN.)	A(IN.)	H(IN.)	DEFLECTION(IN.)	K(LBS./IN.)	K/T	BETA	GAMMA	RHO	U
200.0000	2.0000	5.0000	1.0000	-0.000000549	0.10201E 07	0.91005E 06	100.000	5.00	2.00	0.454460
480.0000	3.2000	8.0000	1.6000	-0.000000550	0.18176E 07	0.56799E 06	150.000	5.00	2.00	0.371065
800.0000	4.0000	10.0000	2.0000	-0.000000608	0.16442E 07	0.41106E 06	200.000	5.00	2.00	0.321351
1080.0000	4.8000	12.0000	2.4000	-0.000000577	0.17322E 07	0.36087E 06	225.000	5.00	2.00	0.302973
1500.0000	6.0000	15.0000	3.0000	-0.000000518	0.19290E 07	0.32151E 06	250.000	5.00	2.00	0.287426
2160.0000	7.2000	18.0000	3.6000	-0.000000527	0.18991E 07	0.26377E 06	300.000	5.00	2.00	0.262382
2800.0000	8.0000	20.0000	4.0000	-0.000000559	0.17880E 07	0.22351E 06	350.000	5.00	2.00	0.242919
3520.0000	8.8000	22.0000	4.4000	-0.000000586	0.17059E 07	0.19385E 06	400.000	5.00	2.00	0.227230
200.0000	2.0000	10.0000	1.0000	-0.000000360	0.27806E 07	0.13903E 07	100.000	10.00	2.00	0.908920
480.0000	3.2000	16.0000	1.6000	-0.000000397	0.25214E 07	0.78795E 06	150.000	10.00	2.00	0.742130
800.0000	4.0000	20.0000	2.0000	-0.000000463	0.21592E 07	0.53980E 06	200.000	10.00	2.00	0.642703
1080.0000	4.8000	24.0000	2.4000	-0.000000448	0.22300E 07	0.46458E 06	225.000	10.00	2.00	0.605946
1500.0000	6.0000	30.0000	3.0000	-0.000000409	0.24424E 07	0.40706E 06	250.000	10.00	2.00	0.574851
2160.0000	7.2000	36.0000	3.6000	-0.000000427	0.23417E 07	0.32523E 06	300.000	10.00	2.00	0.524765
2800.0000	8.0000	40.0000	4.0000	-0.000000463	0.21604E 07	0.27005E 06	350.000	10.00	2.00	0.485838
3520.0000	8.8000	44.0000	4.4000	-0.000000493	0.20281E 07	0.23046E 06	400.000	10.00	2.00	0.454459
200.0000	2.0000	15.0000	1.0000	-0.000000228	0.43894E 07	0.21947E 07	100.000	15.00	2.00	1.363379
480.0000	3.2000	24.0000	1.6000	-0.000000280	0.35674E 07	0.11148E 07	150.000	15.00	2.00	1.113194
800.0000	4.0000	30.0000	2.0000	-0.000000346	0.28875E 07	0.72188E 06	200.000	15.00	2.00	0.964055
1080.0000	4.8000	36.0000	2.4000	-0.000000342	0.29238E 07	0.60913E 06	225.000	15.00	2.00	0.908919
1500.0000	6.0000	45.0000	3.0000	-0.000000317	0.31503E 07	0.52506E 06	250.000	15.00	2.00	0.862277
2160.0000	7.2000	54.0000	3.6000	-0.000000340	0.29434E 07	0.40881E 06	300.000	15.00	2.00	0.787147
2800.0000	8.0000	60.0000	4.0000	-0.000000376	0.26626E 07	0.33282E 06	350.000	15.00	2.00	0.728757
3520.0000	8.8000	66.0000	4.4000	-0.000000406	0.24606E 07	0.27961E 06	400.000	15.00	2.00	0.681689

TABLE 3 RESULTS FOR RADIAL SPRING CONSTANTS FOR $\rho = 4.0$

K=SPRING CONSTANT   BETA=R/T   GAMMA=A/H   RHO=T/H   U=1.81794*A*SQRT(BETA)/R

R(IN.)	T(IN.)	A(IN.)	H(IN.)	DEFLECTION(IN.)	K(LBS./IN.)	K/T	BETA	GAMMA	RHO	U
150.0000	2.0000	2.5000	0.5000	-0.000000515	0.19405E 07	0.97024E 06	75.000	5.00	4.00	0.262383
300.0000	3.0000	3.7500	0.7500	-0.000000464	0.21540E 07	0.71799E 06	100.000	5.00	4.00	0.227230
600.0000	4.0000	5.0000	1.0000	-0.000000528	0.18944E 07	0.47361E 06	150.000	5.00	4.00	0.185532
1000.0000	5.0000	6.2500	1.2500	-0.000000565	0.17700E 07	0.35400E 06	200.000	5.00	4.00	0.160676
1500.0000	6.0000	7.5000	1.5000	-0.000000589	0.16974E 07	0.28290E 06	250.000	5.00	4.00	0.143713
2100.0000	7.0000	8.7500	1.7500	-0.000000606	0.16501E 07	0.23573E 06	300.000	5.00	4.00	0.131191
2800.0000	8.0000	10.0000	2.0000	-0.000000618	0.16170E 07	0.20213E 06	350.000	5.00	4.00	0.121459
3600.0000	9.0000	11.2500	2.2500	-0.000000628	0.15926E 07	0.17696E 06	400.000	5.00	4.00	0.113615
150.0000	2.0000	5.0000	0.5000	-0.000000430	0.23233E 07	0.11616E 07	75.000	10.00	4.00	0.524765
300.0000	3.0000	7.5000	0.7500	-0.000000407	0.24577E 07	0.81924E 06	100.000	10.00	4.00	0.454460
600.0000	4.0000	10.0000	1.0000	-0.000000487	0.20555E 07	0.51387E 06	150.000	10.00	4.00	0.371065
1000.0000	5.0000	12.5000	1.2500	-0.000000534	0.18711E 07	0.37422E 06	200.000	10.00	4.00	0.321352
1500.0000	6.0000	15.0000	1.5000	-0.000000566	0.17663E 07	0.29438E 06	250.000	10.00	4.00	0.287426
2100.0000	7.0000	17.5000	1.7500	-0.000000589	0.16991E 07	0.24273E 06	300.000	10.00	4.00	0.262382
2800.0000	8.0000	20.0000	2.0000	-0.000000605	0.16527E 07	0.20659E 06	350.000	10.00	4.00	0.242919
3600.0000	9.0000	22.5000	2.2500	-0.000000618	0.16188E 07	0.17987E 06	400.000	10.00	4.00	0.227230
150.0000	2.0000	7.5000	0.5000	-0.000000330	0.30290E 07	0.15145E 07	75.000	15.00	4.00	0.787148
300.0000	3.0000	11.2500	0.7500	-0.000000332	0.30157E 07	0.10052E 07	100.000	15.00	4.00	0.681689
600.0000	4.0000	15.0000	1.0000	-0.000000423	0.23668E 07	0.59170E 06	150.000	15.00	4.00	0.556597
1000.0000	5.0000	18.7500	1.2500	-0.000000480	0.20822E 07	0.41645E 06	200.000	15.00	4.00	0.482027
1500.0000	6.0000	22.5000	1.5000	-0.000000520	0.19236E 07	0.32060E 06	250.000	15.00	4.00	0.431138
2100.0000	7.0000	26.2500	1.7500	-0.000000549	0.18229E 07	0.26041E 06	300.000	15.00	4.00	0.393573
2800.0000	8.0000	30.0000	2.0000	-0.000000570	0.17536E 07	0.21920E 06	350.000	15.00	4.00	0.364378
3600.0000	9.0000	33.7500	2.2500	-0.000000587	0.17032E 07	0.18924E 06	400.000	15.00	4.00	0.340845

TABLE 4 RESULTS FOR RADIAL SPRING CONSTANTS FOR $\rho = 10.0$

K=SPRING CONSTANT   BETA=R/T   GAMMA=A/H   RHO=T/H   U=1.81784*A*SQRT(BETA)/R

R(IN.)	T(IN.)	A(IN.)	H(IN.)	DEFLECTION(IN.)	K(LBS./IN.)	K/T	BETA	GAMMA	RHO	U
200.0000	4.0000	2.0000	0.4000	-0.000000173	0.56327E 07	0.14082E 07	50.000	5.00	10.00	0.128541
375.0000	5.0000	2.5000	0.5000	-0.000000213	0.46995E 07	0.93990E 06	75.000	5.00	10.00	0.104953
600.0000	6.0000	3.0000	0.6000	-0.000000236	0.42384E 07	0.70640E 06	100.000	5.00	10.00	0.090892
1050.0000	7.0000	3.5000	0.7000	-0.000000302	0.33089E 07	0.47270E 06	150.000	5.00	10.00	0.074213
1600.0000	8.0000	4.0000	0.8000	-0.000000352	0.28442E 07	0.35553E 06	200.000	5.00	10.00	0.064270
2250.0000	9.0000	4.5000	0.9000	-0.000000390	0.25653E 07	0.28504E 06	250.000	5.00	10.00	0.057485
3000.0000	10.0000	5.0000	1.0000	-0.000000420	0.23793E 07	0.23793E 06	300.000	5.00	10.00	0.052476
3850.0000	11.0000	5.5000	1.1000	-0.000000445	0.22464E 07	0.20422E 06	350.000	5.00	10.00	0.048584
200.0000	4.0000	4.0000	0.4000	-0.000000173	0.57803E 07	0.14451E 07	50.000	10.00	10.00	0.257081
375.0000	5.0000	5.0000	0.5000	-0.000000212	0.47218E 07	0.94436E 06	75.000	10.00	10.00	0.209906
600.0000	6.0000	6.0000	0.6000	-0.000000237	0.42196E 07	0.70327E 06	100.000	10.00	10.00	0.181784
1050.0000	7.0000	7.0000	0.7000	-0.000000306	0.32700E 07	0.46715E 06	150.000	10.00	10.00	0.148426
1600.0000	8.0000	8.0000	0.8000	-0.000000357	0.28038E 07	0.35048E 06	200.000	10.00	10.00	0.128541
2250.0000	9.0000	9.0000	0.9000	-0.000000396	0.25267E 07	0.28075E 06	250.000	10.00	10.00	0.114970
3000.0000	10.0000	10.0000	1.0000	-0.000000427	0.23431E 07	0.23431E 06	300.000	10.00	10.00	0.104953
3850.0000	11.0000	11.0000	1.1000	-0.000000452	0.22125E 07	0.20113E 06	350.000	10.00	10.00	0.097167
200.0000	4.0000	6.0000	0.4000	-0.000000160	0.62594E 07	0.15648E 07	50.000	15.00	10.00	0.385622
375.0000	5.0000	7.5000	0.5000	-0.000000203	0.49300E 07	0.98600E 06	75.000	15.00	10.00	0.314859
600.0000	6.0000	9.0000	0.6000	-0.000000231	0.43283E 07	0.72139E 06	100.000	15.00	10.00	0.272676
1050.0000	7.0000	10.5000	0.7000	-0.000000303	0.32984E 07	0.47120E 06	150.000	15.00	10.00	0.222639
1600.0000	8.0000	12.0000	0.8000	-0.000000356	0.28065E 07	0.35082E 06	200.000	15.00	10.00	0.192811
2250.0000	9.0000	13.5000	0.9000	-0.000000397	0.25187E 07	0.27986E 06	250.000	15.00	10.00	0.172455
3000.0000	10.0000	15.0000	1.0000	-0.000000429	0.23299E 07	0.23299E 06	300.000	15.00	10.00	0.157429
3850.0000	11.0000	16.5000	1.1000	-0.000000455	0.21967E 07	0.19970E 06	350.000	15.00	10.00	0.145751

Required: (a) Spring constant (K_R) at room temperature
(b) Spring constant (K_R) at 600°F

Solution:

$$\beta = \frac{R}{t} = \frac{1600}{8} = 200$$

$$\gamma = \frac{a}{h} = \frac{30}{2} = 15$$

$$\rho = \frac{t}{h} = \frac{8}{2} = 4$$

We refer to Fig. 5 corresponding to $\rho = 4$. On this graph, we select the curve with $\gamma = 15$. For $\beta = 200$, we read from the curve:

$$\frac{K_R}{t} = 0.417 \times 10^6 \text{ lbs/in}^2$$

Substituting $t = 8$ in., we get:

$$K_R = 0.417 \times 10^6 \times 8$$

Hence $K_R = 3.336 \times 10^6$ lb/in.

This value of spring constant (K_R) is at ambient temperature (say 70°F). The spring constant (K_R) at 600°F is obtained as follows:

At T = 70°F, $E_c = 30 \times 10^6$ psi

At T = 600°F, $E_h = 25.7 \times 10^6$ psi

$$K_R \text{ (at 600°F)} = \frac{E_h}{E_c} \cdot K_R \text{ (at ambient temperature)}$$

$$= \frac{25.7}{30.0} \times (3.336 \times 10^6)$$

K_R (at 600°F) = 2.8578×10^6 lb/in.

CONCLUSION

It has been found that the spring constant (K_R) is inversely proportional to radius (R) of the spherical shell and directly proportional to the square of the thickness (t) of the spherical shell.

It can be seen from Tables 1 through 4 that as nozzle radius (a) decreases, the spring constant (K_R) decreases. It can also be shown using Table 1 through 4 that as nozzle thickness (h) decreases, the spring constant (K_R) decreases.

In order to limit the stresses at the juncture of radial nozzle and spherical shell, the design requirement is to reduce the spring constant (K_R) as much as possible. Hence, to achieve a low value of spring constant (K_R), it is recommended:

(i) to increase the spherical shell radius (R)
(ii) to reduce the spherical shell thickness (t)
(iii) to reduce the nozzle radius (a)
(iv) to reduce the nozzle thickness (h)

APPENDIX

DERIVATION OF CONDITIONS FOR VALIDITY OF GRAPHS

To check the validity of graphics, we make the following two derivations:

Derivation No. 1

As suggested by Bijlaard [2], the graphs are applicable only if the deflections are limited to a segment of shell that can be considered as shallow, which still can be assumed if these deflections die out at a distance r of about $0.6R$ from the center of the attachment. The following two values of s are also as per Ref. [2]:

(a) For u (= $a/1$) $\leqslant$ 1.0, Deflections die out at about s (= $r/1$) = 3.5

(b) For u (= $a/1$) $\geqslant$ 1.0, Deflections die out at about s (= $r/1$) = u + 2.5

Since $s = 1.81784 \frac{r}{R}\sqrt{\frac{R}{t}}$, we get:

$$r = 0.55\sqrt{Rt} \cdot s \tag{23}$$

Case (a)

$u \leqslant 1.0$, $s = 3.5$ (For deflections to die out)
From Eq. (23), we get:

$$r = 0.55\sqrt{Rt} \times 3.5$$

Hence
$$r = 1.92\sqrt{Rt} \tag{24}$$

Since $r \leqslant 0.6R$ (For deflection to die out), we get from Eq. (24):

$$1.92\sqrt{Rt} \leqslant 0.6R$$

$$\text{or} \quad \sqrt{\frac{R}{t}} \geqslant \frac{1.92}{0.6} = 3.21$$

$$\text{or} \quad \frac{R}{t} \geqslant 10 \tag{25}$$

Case (b)

$u > 1.0$, $s = u + 2.5$ (For deflections to die out)
From Eq. (23), we get:

$$r = 0.55\sqrt{Rt}\cdot(u + 2.5) \qquad (26)$$

Since $r \leqslant 0.6R$ (For deflections to die out), we get from Eq. (26):

$$0.55\sqrt{Rt}\cdot(u + 2.5) \leqslant 0.6R$$

$$\text{or} \quad \sqrt{\frac{R}{t}} \geqslant (0.9167\,u + 2.292)$$

By approximation, we get:

$$\frac{R}{t} \geqslant (u + 2.3)^2 \qquad (27)$$

Derivation No. 2

As suggested by Bijlaard [2], the graphs may only be expected to be sufficiently accurate if a/R is less than or equal to 1/3.

From Eq. (20) of Ref. [2]:

$$\frac{a}{R} = \left(\frac{u^2\rho}{\gamma}\right)\cdot\frac{1}{[12(1-\mu^2)]^{1/2}} \qquad (28)$$

Note: It can be verified by substituting the values of u, ρ and γ in right hand side of above expression that it is equal to a/r.

Hence from Eq. (28), we get:

$$\left(\frac{u^2\rho}{\gamma}\right)\cdot\frac{1}{[12(1-\mu^2)]^{1/2}} \leqslant \frac{1}{3}$$

Substituting $\mu = 0.3$, we get:

$$u^2 \leqslant 1.101\,\frac{\gamma}{\rho}$$

$$\text{Hence} \quad u \leqslant 1.05\sqrt{\frac{\gamma}{\rho}} \qquad (29)$$

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