ON ROTATIONAL SPRING CONSTANTS AT THE JUNCTURE OF A RADIAL NOZZLE AND A SPHERICAL SHELL

B. Batra* and B. C. Sun, Associate Professor
Mechanical Engineering, New Jersey Institute of Technology
Newark, New Jersey

ABSTRACT

This paper presents the values of the rotational spring constants (i.e. Spring Constants due to bending moment) at the juncture of a radial nozzle and a spherical shell. This rotational spring constant is a function of diameters and thicknesses of nozzle and vessel. It is known that a lower Rotational Spring Constant at the juncture would reduce the local peak stresses which is very important in fatigue design of pressure vessel. This study shows that the value of the Rotational Spring Constant may be reduced by increasing the shell diameter and decreasing the shell thickness, nozzle diameter and nozzle thickness. Analytical derivations for this spring constant are presented in this paper. The actual values of Rotational Spring Constants for various nozzle-shell combination have been computed using a digital computer. These values are tabulated and plotted in this paper to facilitate the vessel design and stress analysis.

NOMENCLATURE

- a = Radius of the middle surface of the radial nozzle
- $A_{1,1}$ to $A_{4,4}$ = Constants
- B_1 to B_4 = Constants
- C_3, C_4, C_{12} = Constants
- $D = Et^3/[12(1-\mu^2)]$ = Flexural Ridigity of spherical shell
- D_1 to D_{12} = Constants
- E_c = Modulus of elasticity at room temperature
- E_h = Modulus of elasticity at temperature of operation
- F = Stress Function
- h = Thickness of the radial nozzle
- H_1 to H_8 = Constants
- K_B = Rotational Spring Constant
- K_1 to K_5 = Constants
- Ker s, Kei s, Ker u, Kei u = Kelvin functions of zero order
- Ker's, Kei's, Ker'u, Kei'u = Derivatives of Kelvin functions
- $l = [R^2t^2/(12(1-\mu^2))]^{1/4}$
- L_1 to L_4 = Constants
- M = Bending moment coming through a nozzle
- M_0 = Bending moment (M_x) in the nozzle at x=0
- M_x = Radial moment acting per unit width upon a normal section of the spherical shell
- M_y = Tangential moment acting per unit width upon a meridional section of the spherical shell
- M_{xy} = Twisting moment in the nozzle or spherical shell
- $N = Eh^3/[12(1-\mu^2)$ Flexural rigidity of nozzle
- N_x = Radial membrane force, acting per unit width upon a normal section of the spherical shell
- N_y = Tangential membrane force, acting per unit width upon a meridional section of the spherical shell
- Q_x = Transverse shear force in cross-section of the nozzle
- $\overline{Q}_x$ = Equivalent transverse shear force for nozzle in section upon which Q_x acts
- Q_{xv} = Transverse shear force in cross-section of the spherical shell
- $\overline{Q}_{xv}$ = Equivalent transverse shear force for spherical shell in section upon which Q_{xv} acts

*Currently Senior Product Design Engineer at General Electric Environmental Services, Inc., Lebanon, Pennsylvania

- R = Radius of middle plane of spherical shell
- r = Radius of curvature of middle surface of Spherical Shell in a latitudinal plane (Refer Fig. 1)
- s = $r/l = 1.81784(r/R).(R/t)^{1/2}$
- t = Thickness of the spherical shell
- T = Temperature of operation
- T_x = Axial membrane force in the nozzle
- T_y = Tangential membrane force in the nozzle
- $T_{x\theta}$ = Membrane shear force in the nozzle
- u = $a/l = 1.81784(a/R).(R/t)^{1/2}$
- u_1 = Axial displacement of the nozzle
- v_1 = Displacement of the nozzle in the tangential. (θ) direction
- v_o = Transverse shear v_x in nozzle at $x=0$
- w_1 = Radial deflection of the nozzle, positive if directed outwards
- w_v = Radial deflection of the spherical shell, positive if directed outwards
- x = Axial coordinate of cylindrical shell (nozzle)
- y = Tangential coordinate of cylindrical and spherical shells
- z = Radial coordinate of cylindrical shell (nozzle)

Greek Symbols

- α = Slope angle at the juncture of spherical shell and nozzle with respect to their original positions due to their deflections
- α_1 = $(1/a)[1-(1/2\mu)+(3(1-\mu^2)\gamma^2+1-(3/4\mu^2))^{1/2}]^{1/2}$
- β = R/t (A Shell Parameter)
- β_o = $[3(1-\mu^2)/(a^2h^2)]^{1/4}$
- β_1 = $(1/a)[-1+(1/2\mu)+(3(1-\mu^2)\gamma^2+1-(3/4\mu^2))^{1/2})^{1/2}$
- γ = a/h (A Nozzle Parameter)
- ε = Unit strain
- θ = Polar coordinate for spherical shell and nozzle, in radians
- μ = Poisson's Ratio (Assumed as 0.3 for calculations)
- ρ = t/h (Shell Thickness/Nozzle Thickness)
- ϕ = Angle between shell axis and normal to middle surface of spherical shell, in radians
- ∇^2 = $(d^2/dr^2)+(1/r)(d/dr)$

INTRODUCTION

With the advent of more and more chemical plants, power plants and other process plants, the safety requirement and economy in design of pressure vessels are gaining high priority. This leads to the requirement of a more accurate stress analysis.

It is known that there exist highly localized stresses at the juncture of nozzle and pressure vessels, both in cylindrical and spherical types. However, these stresses cannot be accurately evaluated even with most modern and highly sophisticated computer programs without reliable values of the spring constants at these junctures. The radial spring constants at the juncture of a radial nozzle and a spherical shell are given in Ref [1]. This paper deals specifically with rotational spring constants (i.e. spring constants due to bending moment) at the juncture of a nozzle and a spherical shell.

Bijlaard has studied the differential equations for a bending moment acting on a spherical shell. His solution leads to the deflection of the spherical shell only [2]. He did not specify the relationships of rotational spring constants in terms of shell parameters (diameter and thickness) and the nozzle parameters (diameter and thickness). This part is explicitly developed here. The solution involves Kelvin functions of zero and first order along with their derivatives. These are obtained by programming the mathematical equations given by Abromowitz and Stegun [3].

DERIVATION OF EXPRESSION FOR DEFLECTION OF SPHERICAL SHELL DUE TO BENDING MOMENT

The radial deflection and stress function for the case of bending moment acting on the nozzle is given by [4]:

$$w_v = (C_3 Ker's + C_4 Kei's)\cos\theta \qquad (1)$$

$$F = [Et^2/(12(1-\mu^2))^{1/2}].(C_3 Kei's - C_4 Ker's + C_{12}s^{-1})\cos\theta \qquad (2)$$

where w_v = Radial deflection of spherical shell (in the direction of exterior normal)

F = Stress function

θ = Polar co-ordinate for cylindrical and spherical shells, in radians

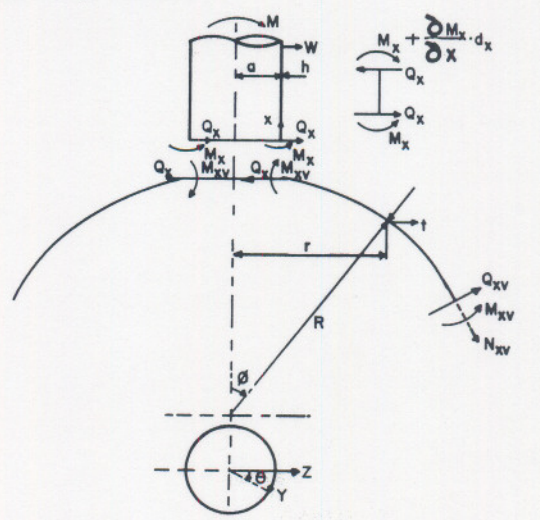

Fig.1 Spherical shell subjected to a bending moment (M) acting on a nozzle

The deflections of the nozzle in axial, tangential and radial directions are given by [2]:

$$u_1 \;=\; [e^{-\alpha_1 x}(H_5 \mathrm{Cos}\ \beta_1 x + H_6 \mathrm{Sin}\ \beta_1 x) - (M/(\pi a h E)).(x/a) - H_3 a].\mathrm{Cos}\ \theta \quad (3)$$

$$v_1 \;=\; [e^{-\alpha_1 x}(H_7 \mathrm{Cos}\ \beta_1 x + H_8 \mathrm{Sin}\ \beta_1 x) - (M/(2\pi a h E)).(x^2/a^2) - H_3 x - H_4\ ] \quad (4)$$

$$w_1 \;=\; [e^{-\alpha_1 x}(H_1 \mathrm{Cos}\ \beta_1 x + H_2 \mathrm{Sin}\ \beta_1 x) + (M/(2\pi a h E)).(x^2/a^2) + 2\mu) + H_3 x + H_4] \quad (5)$$

Equations (1) and (2) contain 3 unknown constants, namely C_3, C_4 and C_{12} while equations (3), (4) and (5) contain 4 unknown constants, namely H_1, H_2, H_3 and H_4. (The constants H_5, H_6, H_7 and H_8 can be expressed in terms of constants H_1 and H_2 as per equations (27) and (40) of Ref. [2]). These 7 unknown constants require 7 boundary conditions between the spherical shell and the nozzle. These are given as follows:

Boundary Condition I:

The cross-section of the nozzle remains circular and its radius does not change. Hence B.C. I is given by:

$$(w_1)_{x=0} = 0 \quad (6)$$

Boundary Condition II:

Since the plane cross-section remains the same, the B.C. II is given by:

$$(u_1)_{x=o} = 0 \quad (7)$$

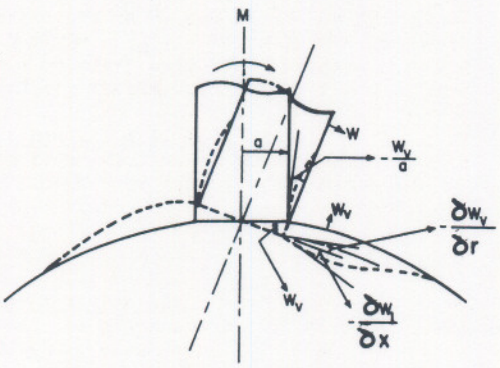

Fig.2 Deflections of nozzle and spherical shell due to a bending moment

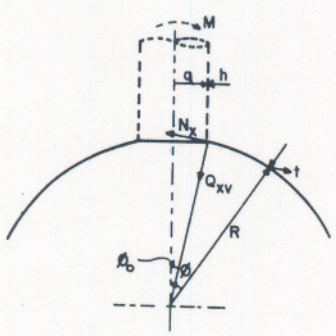

Fig.3 Normal and shear forces acting on the spherical shell at the nozzle-sphrical shell juncture

Boundary Condition III:

Since the shell is subjected to the external moment, we have from Ref. [5]:

$$C_{12} \;=\; [3(1-\mu^2)]^{1/2}.RM/(\pi E t^2 1) \quad (8)$$

Boundary Condition IV:

From Fig. 2, the compatibility of rotation at the juncture of spherical shell and nozzle requires:

$$-(\partial w_v/\partial r) - (\partial w_1/\partial x) = -w_v/a$$

$$\text{or,}\ (w_v/a - \partial w_v/\partial r)_{r=a} = (\partial w_1/\partial x)_{x=0} \quad (9)$$

Boundary Condition V:

The rotation of the spherical shell at r=a and that of nozzle at x=0 should be equal. This leads to:

$$(\varepsilon_{yv})_{r=a} = (\varepsilon_\theta)_{x=0} \quad (10)$$

Boundary Condition VI:

At the juncture, the bending moment M_x in the walls of the nozzle and shell should be equal. Hence, we get:

$$(M_{xv})_{r=a} = (M_x)_{x=0} \quad (11)$$

Boundary Condition VII:

The shear force Q_x in the nozzle at x=0 has to be in equilibrium with the horizontal components of the force in the shell at r=a. Hence from Fig. 1 and Fig. 3, we have:

$$N_x \mathrm{Cos}\phi_o + \bar{Q}_{xv} \mathrm{Sin}\phi_o = \bar{Q}_x$$

Here $\bar{Q}_{xv}$ and $\bar{Q}_x$ are the equivalent transverse shear forces for shell and nozzle respectively, including the effects of the twisting moments. Since ϕ_o is assumed to be small, this condition reduces to:

$$(1 - a^2/2R^2)N_x + (a/R)\bar{Q}_{xv} = (Q_x)_{x=0} \quad (12)$$

Since boundary condition-III determine C_{12}, the rest six unknowns H_1, H_2, H_3, H_4, C_3 and C_4 are to be found using conditions I, II and IV through VII.

From conditions I & II, we get [2]:

$$H_4 \;=\; -H_1 - (\mu M/\pi a h E) \quad (13)$$

$$\text{and}\ H_3 \;=\; H_5/a \quad (14)$$

Using the remaining 4 conditions (i.e. IV through VII), the constants C_3, C_4, H_1 and H_2 can be found from equations (54), (59), (61) and (68) of Ref. [2]. These equations are numbered (15) to (18) in this paper and are given below:

$$D_1C_3 + D_2C_4 - [\alpha_1 - (L_1/a)]H_1 - [\beta_1 + (L_2/a)]H_2 = 0 \quad (15)$$

$$D_3C_3 + D_4C_4 - [(1+L_3)\gamma(12(1-\mu^2))^{1/2}/\rho u^2]H_1 - [L_4\gamma(12(1-\mu^2))^{1/2}/\rho u^2]H_2 = (1/\pi\mu)[(\mu\rho^2/\gamma) - ((1+\mu).(12(1-\mu^2))^{1/2}/u^2)].(RM/Et^2 1) \quad (16)$$

$$D_5C_3 + D_6C_4 - D_7H_1 + D_8H_2 = [(2-\mu^2)/(12(1-\mu^2)\pi)].(RM/Et^2 1) \quad (17)$$

$$D_9C_3+D_{10}C_4+D_{11}H_1-D_{12}H_2=[(1-\eta)(12(1-\mu^2))^{1/2}/\pi u^3].(RM/Et^2 1) \quad (18)$$

where constants D_1 to D_{12} are given below:

$$D_1 = (u/a).[Kei\ u + (2/u)Ker'u] \quad (19)$$

$$D_2 = (u/a)[(2/u).Kei'u - Ker\ u] \quad (20)$$

$$D_3 = -[(1+\mu)/u].Ker\ u + Ker'u + [2(1+\mu/u^2].Kei'u \quad (21)$$

$$D_4 = -[(1+\mu)/u].Kei\ u + Kei'u - [2(1+\mu)u^2].Kei'u \quad (22)$$

$$D_5 = [\gamma^2 u/(12(1-\mu^2))^{1/2}].[(1-\mu)u^{-1}.Kei\ u + 2(1-\mu)u^{-2}.Ker'u - Kei'u] \quad (23)$$

$$D_6 = [\gamma^2 u/(12(1-\mu^2))^{1/2}].[-(1-\mu)u^{-1}.Ker\ u + 2(1-\mu)u^{-2}.Kei'u + Ker'u] \quad (24)$$

$$D_7 = [\gamma^2 a^2/\rho^3 u(12(1-\mu^2))^{1/2}].[\alpha_1^2-\beta_1^2-\mu(1+L_3)/a^2 + (\alpha_1 L_1+\beta_1 L_2)/a] \quad (25)$$

$$D_8 = [\gamma^2 a^2/(\rho^3 u(12(1-\mu^2))^{1/2}].[2\alpha_1\beta_1 + \mu L_4/a^2-(\alpha_1 L_2-\beta_1 L_1)/a] \quad (26)$$

$$D_9 = (1-\eta)u^{-1}(Ker\ u-2u^{-1}Kei'u) - [u^3\rho^2/(12(1-\mu^2)\gamma^2)].[(1-\mu)u^{-2}\ Kei\ u - Ker\ u+u^{-1}.Kei'u + 2(1-\mu).u^{-3}.Ker'u] \quad (27)$$

$$D_{10} = (1-\eta).u^{-1}.(Kei\ u+2u^{-1}.Ker'u) + [u^3\rho^2/(12(1-\mu^2))\gamma^2].[(1-\mu)u^{-2}.\ Ker\ u + Kei\ u+u^{-1}Ker'u-2(1-\mu).u^{-3}.Kei'u] \quad (28)$$

$$D_{11} = [a^3/(u^2\rho^2\gamma(12(1-\mu^2))^{1/2}].[\alpha_1(\alpha_1^2-3\beta_1^2) - (2-\mu)\alpha_1/a^2+(3-\mu)(-\alpha_1 L_3-\beta_1 L_4)/(2a^2) + (\alpha_1^2-\beta_1^2)L_1+2\alpha_1\beta_1 L_2)/a+(1-\mu)L_1/(2a^3)] \quad (29)$$

$$D_{12} = [a^3/u^2\rho^2\gamma(12(1-\mu^2))^{1/2}].[\beta_1(3\alpha^2-\beta_1^2) - (2-\mu)\beta_1/a^2-(3-\mu)(-\alpha_1 L_4+\beta_1 L_3)/(2a^2) - ((\alpha_1^2-\beta_1^2)L_2-2\alpha_1\beta_1 L_1)/a-(1-\mu)L_2/(2a^3)] \quad (30)$$

where α_1 and β_1 are given by Eq. (11) of Ref. [2] and are as follows:

$$\alpha_1 = (1/a).[1-1/(2\mu)+(3(1-\mu^2)\gamma^2+1-3/(4\mu^2))^{1/2}]^{1/2} \quad (31)$$

$$\beta_1 = (1/a).[-(1-(1/2\mu))+(3(1-\mu^2)\gamma^2+1-3/(4\mu^2))^{1/2}]^{1/2} \quad (32)$$

The constants L_1 to L_4 are as follows:

$$L_1 = K_1/K_5 \quad (33)$$

$$L_2 = K_2/K_5 \quad (34)$$

$$L_3 = K_3/K_5 \quad (35)$$

$$L_4 = K_4/K_5 \quad (36)$$

where constants K_1 to K_5 are as given below:

$$K_1 = a\alpha_1[(1-\mu).(1-3\mu(1+\mu+\mu^2))-12(1-\mu^4)\gamma^2 + (2(1-\mu).(2+3\mu+3\mu^2)+24\mu(1-\mu^2)\gamma^2)a^2\beta_1^2] \quad (37)$$

$$K_2 = -a\beta_1[(1-\mu).(1-3\mu(1+\mu+\mu^2))-12(1-\mu^4)\gamma^2 - (2(1-\mu).(2+3\mu+3\mu^2)+24\mu(1-\mu^2)\gamma^2)a^2\alpha_1^2] \quad (38)$$

$$K_3 = (1-\mu)(3+\mu)(1-3\mu^2)+12(1-2\mu-\mu^2)(1-\mu^2)\gamma^2 \quad (39)$$

$$K_4 = 2[(4+9\mu+3\mu^2)(1-\mu)+12(2+\mu)(1-\mu^2)\gamma^2]a^2\alpha_1\beta_1 \quad (40)$$

$$K_5 = [12(1-\mu^2)\gamma^2+(1-\mu).(1+3\mu)]^2 \quad (41)$$

Thus the final solution for the deflection of the spherical shell due to a bending moment M is given by equation (1) where constants C_3 and C_4 are obtained by solution of 4 simultaneous equations (15) to (18) for 4 unknowns; namely C_3, C_4, H_1 and H_2. The other constants; namely D_1 to D_{12}, L_1 to L_4, K_1 to K_5, α_1 and β_1 used in these equations are given by equations (19) to (41).

APPROACH FOR EVALUATION OF ROTATIONAL SPRING CONSTANTS

The expression for deflection

$$w_v = (C_3 Ker's + C_4 Kei's)\cos\theta \quad (1)$$

is valid at any point of spherical pressure vessel specified by radius r. Since, we are interested only in the deflection of the vessel at the juncture of nozzle and spherical shell, we let r=a.

At r=a, s=a/l=u

Also, for present analysis $\theta=0$ since we are interested in maximum deflection due to moment which occurs in the plane where the moment is acting. Hence, the expression for w_v for this case becomes:

$$w_v = C_3 Ker'u + C_4 Kei'u \quad (42)$$

where $u = a/l = a.[12(1-\mu^2)/R^2t^2]^{1/4}$

Substituting $\mu=0.3$, we get:

$$u = 1.81784.a/\sqrt{Rt} \quad \text{or}$$

$$u = 1.81784.(a/R)\sqrt{R/t} \quad (43)$$

The rotational spring constant due to bending moment is given by referring to Fig. 4:

$$K_B = M/|\phi| \quad (44)$$

From Fig. 4:

$$w_v = \phi.a$$

$$|\phi| = |w_v|/a$$

Substituting this expression for ϕ in equation (44), we get:

$$K_B = M.a/|w_v| \quad (45)$$

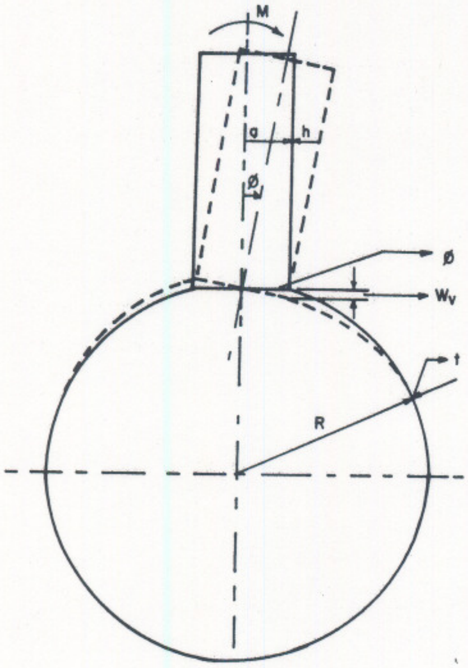

Fig.4 Deflection (Wv) due to a bending moment (M)

Taking M = 1.0 in-lb. and substituting the expression for w_v from equation (42), the expression for rotational spring constant becomes:

$$K_B = a/(C_3 Ker'u + C_4 Kei'u) \tag{46}$$

For a given set of shell parameters (R & t) and nozzle parameters (a & h), unique values of u, C_3 and C_4 exist. Hence, given R,t,a and h, unique values of deflection (w_v) and rotational spring constant (K_B) can be obtained. For convenience, we define 3 independent dimensionless parameters β,γ and ρ as follows:

Shell Parameter: $\beta = R/t$
Nozzle Parameter: $\gamma = a/h$
Shell to nozzle thickness ratio: $\rho = t/h$

Fixing the values of β,γ and ρ does not fix R, t,a and h e.g. If R,t,a and h are doubled, the ratios β,γ and ρ still remain the same while the rotational spring constant (K_B) for this new set of values gets changed. Hence, to arrive at a unique value of K_B, we must fix one more parameter apart from β,γ and ρ. This is achieved as follows:

For given values of R,t,a and h, we find the value of u. Since, we are interested at the juncture of shell and nozzle, using u=s curve [2], we read a constant value for $w_v E.t^2/(M\sqrt{R/t}.\cos\theta)$.

$$\text{Hence,} \quad \frac{w_v E.t^2}{M\sqrt{R/t}.\cos\theta} = C \text{ (Constant)}$$

For our case θ=0. Hence we have:

$$\frac{w_v E.t^2}{M\sqrt{R/t}} = C$$

$$\frac{M}{w_v} = \frac{E.t^2}{C.\sqrt{R/t}} \tag{47}$$

From equation (45), we have the spring constant (K_B) as follows:

$$K_B = \frac{M.a}{|w_v|} = \frac{E.t^2.a}{C.\sqrt{R/t}}$$

$$K_B = E.t^2.a/(C.\sqrt{R/t}) \tag{48}$$

where E is the Young's Modulus.

Equation (48) can also be written as:

$$K_B/(a.t^2) = C'/\sqrt{\beta} \tag{49}$$

where β = R/t (as per definition)

and C' = E/C = Another Constant

Hence, to fix the value of K_B for fixed values of β,γ and ρ, we must fix a and t also.

It logically follows from equation (49) that there exists a unique value of $K_B/(a.t^2)$ for a given set of values β,γ and ρ.

At this stage, it seems logical to plot the values of $K_B/a.t^2$ vs. β for various combinations of γ and ρ because there exists a unique value of $K_B/(a.t^2)$ for a given set of β,γ, and ρ. In fact, in latter part of this paper, values for $K_B/(a.t^2)$ have been plotted against the shell parameter β(R/t) for 3 different values of the nozzle parameter γ(a/h) and unique values of the constant ρ(t/h).

Using all the above equations, a computer program has been written to evaluate spring constants (K_B) and ratios $K_B/(a.t^2)$ for various combinations of R,t,a & h (i.e. β,γ and ρ). The various equations for Kelvin functions (ker u, Kei u) and their derivatives (Ker´u, Kei´u) are taken from Ref. [3].

Computer results for spring constants K_B and ratio $K_B/(a.t^2)$ are obtained for various values of β,γ and ρ. Tables 1 through 4 give ranges for ρ=1.0,2.0,4.0,10.0; γ=5,10,15 and various ranges of β.

The values of deflection (w_v) computed in this study have been verified with Bijlaard's work in Ref. [2] and are found to be matching quite closely. Values taken from Tables 1 through 4 are also plotted in this paper to facilitate the vessel design and stress analysis. These are given in Fig. 5 to 8 respectively.

One must note that the values of rotational spring constants are valid only if the deflections are limited to a segment of shell that can be considered shallow. This leads to conditions:

(1) $u \le 1.0$, $R/t \ge 10$
(2) $u > 1.0$, $R/t \ge (u + 2.3)^2$

Also the values of spring constants are considered accurate only if a/R is less than or equal to 1/3. This leads to the condition:

(3) $a/R \le 1/3$, i.e. $u \le 1.05\sqrt{\gamma/\rho}$

The above three conditions are derived in Ref. [4] and can be found in Appendix I. The computer program is designed to take care of these 3 conditions.

CORRECTION OF THE SPRING CONSTANTS FOR HOT MODULUS

As we know the value of E varies depnding upon the temperature of operation. The graphs plotted here are also for cold condition i.e. E=30E06 psi.

In case, the temperature of operation is higher than the normal ambient temperature, the value of bending spring constant (K_B as obtained at ambient temperature) must be corrected as follows:

As given by equation (48), we know:

$$K_B = E.t^2.a/(C.\sqrt{R/t})$$

For a given geometry of spherical shell:

$$K_B \;\alpha\; E$$

$$\frac{K_B \text{ (at temperature of operation)}}{K_B \text{ (at ambient temperature)}} = \frac{E_h}{E_c}$$

or K_B (at temperature of operation =

$(E_h/E_c).K_B$(at ambient temperature)

E_c = 30E06

E_h = Modulus of Elasticity at temperature of operation

Thus, if β,γ,ρ,t,a and temperature of operation are specified, the spring constant can readily be found using these curves. Given below is an example to illustrate the use of these curves:

NUMERICAL EXAMPLE

Given:

R = 500" , t = 2" , a = 5" , h = 1"

Temperature of operation = 600 F

Material : C-Steel with Carbon Content $\leq$ 0.3%

Required:

(a) Rotational spring constant (K_B) at room temperature

(b) Rotational spring constant (K_B) at 600 F

Solution:

β = (R/t) = 500/2 = 250

γ = (a/h) = 5/1 = 5

ρ = (t/h) = 2/1 = 2

We refer to Fig. 6 corresponding to the value of ρ = 2. On this graph, we select the curve with γ = 5. For β = 250, we read from the curve:

$$K_B/(a.t^2) = 0.675E07 \text{ lbs/in}^2$$

Substituting a = 5" and t = 2", we set:

$$K_B = 0.675E07x5x2^2 = 13.5E07 \text{ in-lb/rad.}$$

This value of rotational spring constant (K_B) is at ambient temperature (say 70 F). The bending spring constant (K_B) at 600 F is obtained as follows:

At T = 70 F , E_c = 30E06 psi

T = 600F , E_h = 25.7E06 psi

K_B (at 600 F) = $(E_h/E_c).K_B$(at ambient temperature)

= (25.7/30.0)x13.5E07

K_B (at 600 F) = 11.57E07 in-lb/rad.

CONCLUSION

It has been found that rotational spring constant (K_B) is inversely proportional to square root of the radius of the spherical shell and directional proportional to $t^{5/2}$ (where t is the thickness of the spherical shell) and directly proportional to nozzle radius (a).

It can be seen from Tables 1 through 4 that as nozzle thickness (h) decreases, the bending spring constant (K_B) decreases.

In order to limit the stresses at the juncture of radial nozzle and spherical shell, the design requirement is to reduce the rotational spring constant (K_B). Hence, to achieve a low value of rotational spring constant (K_B), it is recommended:

(1) To increase the spherical shell radius (R)
(2) To reduce the spherical shell thickness (t)
(3) To reduce the nozzle radius (a)
(4) To reduce the nozzle thickness (h)

REFERENCES

1. Batra, B. & Sun, B.C., "On Radial Spring Constants at the juncture of a radial nozzle and a spherical shell". Paper presented at Fifth International Conference on Pressure Vessel Technology held in San Francisco, CA during September 1984.

2. Bijlaard, P.P., "Stresses in a spherical vessel from external moments acting on a pipe", Welding research Council Bulletin No. 49, pp. 31-62 (April 1959).

3. Milton Abramowitz & Irene A. Stegun, "Handbook of Mathematical Functions", pp. 379-380 (Dec. 1965, 4th. Print).

4. Batra, B., "Evaluation of spring constants at the juncture of a radial nozzle and a spherical shell due to radial load and bending moment". M.S. Thesis (1983), New Jersey Institute of Technology, Newark, NJ.

5. Reissner, E., "Stresses and small displacements of shallow spherical shells". International Joural of Mathematical Physics, 25, pp. 80-85 (1946).

6. Bijlaard, P.P., "Stresses in a spherical vessel from radial loads acting on a pipe". Welding Research Council Bulletin No. 49, pp. 1-30 (April 1959).

APPENDIX

DERIVATION OF CONDITIONS FOR VALIDITY OF GRAPHS:

To check the validity of graphs, we make the following 2 derivations:

Derivation I:

As suggested by Bijlaard [6], the graphs are applicable only if deflections are limited to a segment of shell that can be considered as shallow, which still can be assumed if these deflections die out at a distance r of about 0.6R from the center of the attachment. The following two values of s are also as per Ref. [6]:

(a) For $u(a/l) \leq 1.0$, Deflections die out at about $s(r/l) = 3.5$
(b) For $u(a/l) > 1.0$, Deflections die out at about $s(r/l) = u+2.5$

Since $s = 1.81784.(r/R).\sqrt{R/t}$, we get

$$r = 0.55 \sqrt{Rt}.s \qquad (50)$$

Case (a)

$u \leq 1.0$, $s=3.5$ (For deflections to die out)

From Eq. (50), we get:

$$r = 0.55\sqrt{R/t} \times 3.5$$

$$r = 1.92\sqrt{R/t} \qquad (51)$$

Since $r \leq 0.6R$ (For deflection to die out), we set from Eq. (51):

$$1.92\sqrt{R/t} \leq 0.6R$$

$$\sqrt{R/t} \geq 1.92/0.6 = 3.21$$

or $$R/t \geq 10 \qquad (52)$$

Case (b)

$u > 1.0$, $s=u+2.5$ (For deflections to die out)

From Eq. (50), we get

$$r = 0.55\sqrt{R/t}.(u+2.5) \qquad (53)$$

Since $r \leq 0.6R$ (For deflections to die out), we get from Eq. (53):

$$0.55\sqrt{R/t}.(u+2.5) \leq 0.6R$$

or $$\sqrt{R/t} \geq (0.9167u+2.292)$$

By approximation, we set:

$$R/t \geq (u+2.3)^2 \qquad (54)$$

Derivation II:

As suggested by Bijlaard [6], the graphs may only be expected to be sufficiently accurate if a/R is less than or equal to 1/3.

From Eq. (20) of Ref. [6]:

$$a/R = (u^2.\rho/\gamma)/[12(1-\mu^2)]^{\frac{1}{2}} \qquad (55)$$

(Note: It can be verified by substituting the values of u, ρ & γ in right side of above equation that it is equal to a/R)

Hence from Eq. (55), we get:

$$(u^2.\rho/\gamma)/[12(1-\mu^2)]^{\frac{1}{2}} \leq 1/3$$

Substituting $\mu=0.3$, we get:

$$u^2 \leq 1.101.(\gamma/\rho)$$

$$u \leq 1.05\sqrt{\gamma/\rho} \qquad (56)$$

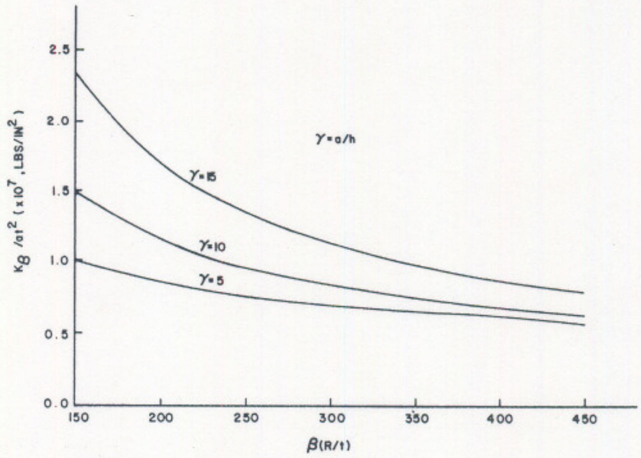

Fig.5 Graph for rotational spring constants for $\rho(t/h) = 1.0$

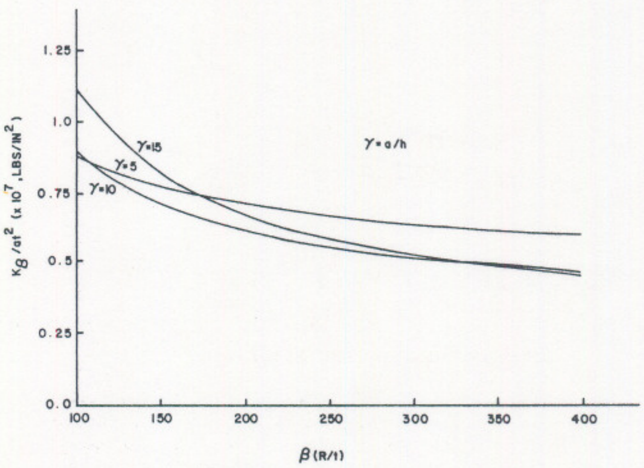

Fig.6 Graph for rotational apring constants for $\rho(t/h) = 2.0$

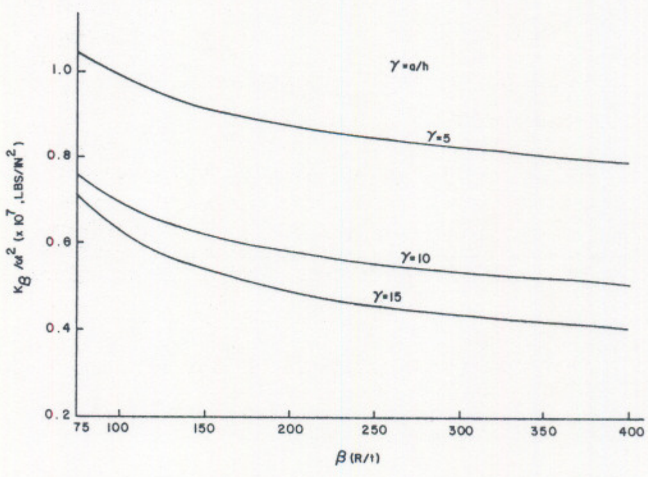

Fig.7 Graph for rotational spring constants for $\rho(t/h) = 4.0$

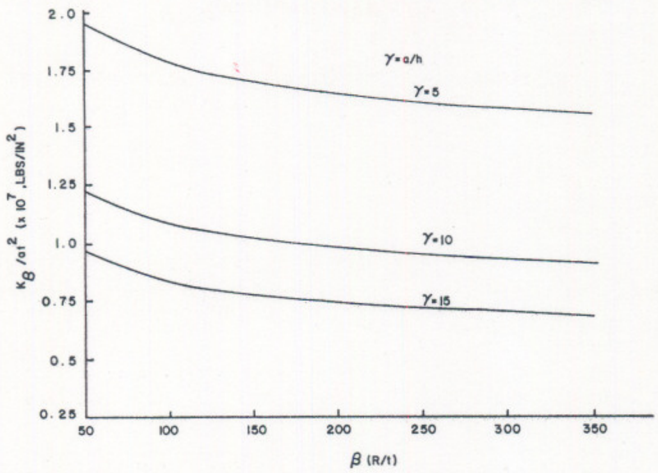

Fig. 8 Graph for rotational spring constants for $\rho(t/h) = 10.0$

TABLE 1: RESULTS FOR ROTATIONAL SPRING CONSTANTS FOR RHO = 1.0

K_B=SPRING CONSTANT   BETA= R/t   GAMMA= a/h   RHO= t/h   u=1.81784* a *SQRT(BETA)/R

R	t	a	h	DEFL.	K_B	K_B/(a*t**2)	BETA	GAMMA	u
150.	1.0	5.0	1.0	-0.000000098	0.50934E 08	0.10187E 08	150.	5.0	0.742130
320.	1.6	8.0	1.6	-0.000000045	0.17741E 09	0.86628E 07	200.	5.0	0.642703
500.	2.0	10.0	2.0	-0.000000032	0.30897E 09	0.77243E 07	250.	5.0	0.574851
720.	2.4	12.0	2.4	-0.000000025	0.48931E 09	0.70792E 07	300.	5.0	0.524764
1050.	3.0	15.0	3.0	-0.000000017	0.89148E 09	0.66035E 07	350.	5.0	0.485838
1440.	3.6	18.0	3.6	-0.000000012	0.14546E 10	0.62353E 07	400.	5.0	0.454459
1800.	4.0	20.0	4.0	-0.000000011	0.19008E 10	0.59399E 07	450.	5.0	0.428469
2200.	4.4	22.0	4.4	-0.000000009	0.24262E 10	0.56964E 07	500.	5.0	0.406481
150.	1.0	10.0	1.0	-0.000000066	0.15218E 09	0.15218E 08	150.	10.0	1.484259
320.	1.6	16.0	1.6	-0.000000033	0.48058E 09	0.11733E 08	200.	10.0	1.285406
500.	2.0	20.0	2.0	-0.000000026	0.77936E 09	0.97419E 07	250.	10.0	1.149702
720.	2.4	24.0	2.4	-0.000000021	0.11683E 10	0.84512E 07	300.	10.0	1.049528
1050.	3.0	30.0	3.0	-0.000000015	0.20369E 10	0.75440E 07	350.	10.0	0.971676
1440.	3.6	36.0	3.6	-0.000000011	0.32050E 10	0.68695E 07	400.	10.0	0.908919
1800.	4.0	40.0	4.0	-0.000000010	0.40620E 10	0.63470E 07	450.	10.0	0.856937
2200.	4.4	44.0	4.4	-0.000000009	0.50507E 10	0.59292E 07	500.	10.0	0.812962
150.	1.0	15.0	1.0	-0.000000042	0.35928E 09	0.23952E 08	150.	15.0	2.226388
320.	1.6	24.0	1.6	-0.000000022	0.10670E 10	0.17367E 08	200.	15.0	1.928108
500.	2.0	30.0	2.0	-0.000000018	0.16501E 10	0.13751E 08	250.	15.0	1.724553
720.	2.4	36.0	2.4	-0.000000015	0.23805E 10	0.11480E 08	300.	15.0	1.574293
1050.	3.0	45.0	3.0	-0.000000011	0.40197E 10	0.99253E 07	350.	15.0	1.457513
1440.	3.6	54.0	3.6	-0.000000009	0.61554E 10	0.87955E 07	400.	15.0	1.363378
1800.	4.0	60.0	4.0	-0.000000008	0.76200E 10	0.79375E 07	450.	15.0	1.285405
2200.	4.4	66.0	4.4	-0.000000007	0.92812E 10	0.72636E 07	500.	15.0	1.219442

UNITS : R (IN.) a (IN.) DEFL. (IN.)
t (IN.) h (IN.) K_B (IN.-LB/RAD.)

TABLE 2: RESULTS FOR RATATIONAL SPRING CONSTANTS FOR RHO = 2.0

K_B=SPRING CONSTANT BETA= R/t GAMMA= a/h RHO= t/h u =1.81784* a *SQRT(BETA)/R

R	t	a	h	DEFL.	K_B	K_B/(a*t**2)	BETA	GAMMA	u
200.	2.0	5.0	1.0	-0.000000028	0.17716E 09	0.88579E 07	100.	5.0	0.454460
480.	3.2	8.0	1.6	-0.000000013	0.63614E 09	0.77654E 07	150.	5.0	0.371065
800.	4.0	10.0	2.0	-0.000000009	0.11436E 10	0.71476E 07	200.	5.0	0.321351
1080.	4.8	12.0	2.4	-0.000000006	0.19143E 10	0.69237E 07	225.	5.0	0.302973
1500.	6.0	15.0	3.0	-0.000000004	0.36372E 10	0.67356E 07	250.	5.0	0.287426
2160.	7.2	18.0	3.6	-0.000000003	0.60044E 10	0.64347E 07	300.	5.0	0.262382
2800.	8.0	20.0	4.0	-0.000000003	0.79382E 10	0.62017E 07	350.	5.0	0.242919
3520.	8.8	22.0	4.4	-0.000000002	0.10246E 11	0.60139E 07	400.	5.0	0.227230
200.	2.0	10.0	1.0	-0.000000027	0.36402E 09	0.91006E 07	100.	10.0	0.908920
480.	3.2	16.0	1.6	-0.000000014	0.11825E 10	0.72173E 07	150.	10.0	0.742130
800.	4.0	20.0	2.0	-0.000000010	0.20034E 10	0.62606E 07	200.	10.0	0.642703
1080.	4.8	24.0	2.4	-0.000000007	0.32813E 10	0.59340E 07	225.	10.0	0.605946
1500.	6.0	30.0	3.0	-0.000000005	0.61216E 10	0.56681E 07	250.	10.0	0.574851
2160.	7.2	36.0	3.6	-0.000000004	0.98136E 10	0.52585E 07	300.	10.0	0.524765
2800.	8.0	40.0	4.0	-0.000000003	0.12684E 11	0.49547E 07	350.	10.0	0.485838
3520.	8.8	44.0	4.4	-0.000000003	0.16077E 11	0.47184E 07	400.	10.0	0.454459
200.	2.0	15.0	1.0	-0.000000022	0.68538E 09	0.11423E 08	100.	15.0	1.363379
480.	3.2	24.0	1.6	-0.000000012	0.20405E 10	0.83029E 07	150.	15.0	1.113194
800.	4.0	30.0	2.0	-0.000000009	0.32712E 10	0.68150E 07	200.	15.0	0.964055
1080.	4.8	36.0	2.4	-0.000000007	0.52468E 10	0.63257E 07	225.	15.0	0.908919
1500.	6.0	45.0	3.0	-0.000000005	0.96150E 10	0.59352E 07	250.	15.0	0.862277
2160.	7.2	54.0	3.6	-0.000000004	0.14973E 11	0.53487E 07	300.	15.0	0.787147
2800.	8.0	60.0	4.0	-0.000000003	0.18918E 11	0.49264E 07	350.	15.0	0.728757
3520.	8.8	66.0	4.4	-0.000000003	0.23541E 11	0.46060E 07	400.	15.0	0.681689

UNITS : R (IN.) a (IN.) DEFL. (IN.)
t (IN.) h (IN.) K_B (IN.-LB/RAD.)

TABLE 3: RESULTS FOR ROTATIONAL SPRING CONSTANTS FOR RHO = 4.0

K_B =SPRING CONSTANT BETA=R/ t GAMMA= a/h RHO= t/h u=1.81784* a *SQRT(BETA)/R

R	t	a	h	DEFL.	K_B	K_B/(a*t**2)	BETA	GAMMA	u
150.	2.0	2.50	0.50	-0.000000024	0.10523E 09	0.10523E 08	75.0	5.0	0.262383
300.	3.0	3.75	0.75	-0.000000011	0.33525E 09	0.99335E 07	100.0	5.0	0.227230
600.	4.0	5.00	1.00	-0.000000007	0.73759E 09	0.92198E 07	150.0	5.0	0.185532
1000.	5.0	6.25	1.25	-0.000000005	0.13718E 10	0.87793E 07	200.0	5.0	0.160676
1500.	6.0	7.50	1.50	-0.000000003	0.22864E 10	0.84683E 07	250.0	5.0	0.143713
2100.	7.0	8.75	1.75	-0.000000002	0.35292E 10	0.82314E 07	300.0	5.0	0.131191
2800.	8.0	10.00	2.00	-0.000000002	0.51469E 10	0.80421E 07	350.0	5.0	0.121459
3600.	9.0	11.25	2.25	-0.000000002	0.71855E 10	0.78854E 07	400.0	5.0	0.113615
150.	2.0	5.00	0.50	-0.000000033	0.15328E 09	0.76640E 07	75.0	10.0	0.524765
300.	3.0	7.50	0.75	-0.000000016	0.47154E 09	0.69858E 07	100.0	10.0	0.454460
600.	4.0	10.00	1.00	-0.000000010	0.99771E 09	0.62357E 07	150.0	10.0	0.371065
1000.	5.0	12.50	1.25	-0.000000007	0.18150E 10	0.58078E 07	200.0	10.0	0.321352
1500.	6.0	15.00	1.50	-0.000000005	0.29810E 10	0.55204E 07	250.0	10.0	0.287426
2100.	7.0	17.50	1.75	-0.000000004	0.45524E 10	0.53089E 07	300.0	10.0	0.262382
2800.	8.0	20.00	2.00	-0.000000003	0.65845E 10	0.51442E 07	350.0	10.0	0.242919
3600.	9.0	22.50	2.25	-0.000000002	0.91318E 10	0.50106E 07	400.0	10.0	0.227230
150.	2.0	7.50	0.50	-0.000000035	0.21716E 09	0.72388E 07	75.0	15.0	0.787148
300.	3.0	11.25	0.75	-0.000000018	0.64074E 09	0.63283E 07	100.0	15.0	0.681689
600.	4.0	15.00	1.00	-0.000000012	0.12937E 10	0.53906E 07	150.0	15.0	0.556597
1000.	5.0	18.75	1.25	-0.000000008	0.22927E 10	0.48911E 07	200.0	15.0	0.482027
1500.	6.0	22.50	1.50	-0.000000006	0.37021E 10	0.45705E 07	250.0	15.0	0.431138
2100.	7.0	26.25	1.75	-0.000000005	0.55854E 10	0.43424E 07	300.0	15.0	0.393573
2800.	8.0	30.00	2.00	-0.000000004	0.80047E 10	0.41691E 07	350.0	15.0	0.364378
3600.	9.0	33.75	2.25	-0.000000003	0.11021E 11	0.40315E 07	400.0	15.0	0.340845

UNITS : R (IN.) a (IN.) DEFL. (IN.)
t (IN.) h (IN.) K_B(IN.-LB/RAD.)

TABLE 4: RESULTS FOR ROTATIONAL SPRING CONSTANTS FOR RHO = 10.0

K_B =SPRING CONSTANT BETA=R/t GAMMA= a/h RHO=t/h u =1.81784*a *SQRT(BETA)/R

R	t	a	h	DEFL.	K_B	K_B/(a*t**2)	BETA	GAMMA	u
200.	4.0	2.0	0.4	-0.000000003	0.63080E 09	0.19712E 08	50.	5.0	0.128541
375.	5.0	2.5	0.5	-0.000000002	0.11631E 10	0.18609E 08	75.	5.0	0.104953
600.	6.0	3.0	0.6	-0.000000002	0.19335E 10	0.17903E 08	100.	5.0	0.090892
1050.	7.0	3.5	0.7	-0.000000001	0.29152E 10	0.16998E 08	150.	5.0	0.074213
1600.	8.0	4.0	0.8	-0.000000001	0.42015E 10	0.16412E 08	200.	5.0	0.064270
2250.	9.0	4.5	0.9	-0.000000001	0.58266E 10	0.15985E 08	250.	5.0	0.057485
3000.	10.0	5.0	1.0	-0.000000001	0.78265E 10	0.15653E 08	300.	5.0	0.052476
3850.	11.0	5.5	1.1	-0.000000001	0.10237E 11	0.15383E 08	350.	5.0	0.048584
200.	4.0	4.0	0.4	-0.000000005	0.78719E 09	0.12300E 08	50.	10.0	0.257081
375.	5.0	5.0	0.5	-0.000000004	0.14262E 10	0.11410E 08	75.	10.0	0.209906
600.	6.0	6.0	0.6	-0.000000003	0.23466E 10	0.10864E 08	100.	10.0	0.181784
1050.	7.0	7.0	0.7	-0.000000002	0.34940E 10	0.10187E 08	150.	10.0	0.148426
1600.	8.0	8.0	0.8	-0.000000002	0.49972E 10	0.97601E 07	200.	10.0	0.128541
2250.	9.0	9.0	0.9	-0.000000001	0.68926E 10	0.94549E 07	250.	10.0	0.114970
3000.	10.0	10.0	1.0	-0.000000001	0.92203E 10	0.92203E 07	300.	10.0	0.104953
3850.	11.0	11.0	1.1	-0.000000001	0.12021E 11	0.90314E 07	350.	10.0	0.097167
200.	4.0	6.0	0.4	-0.000000006	0.93747E 09	0.97653E 07	50.	15.0	0.385622
375.	5.0	7.5	0.5	-0.000000005	0.16655E 10	0.88826E 07	75.	15.0	0.314859
600.	6.0	9.0	0.6	-0.000000003	0.27102E 10	0.83648E 07	100.	15.0	0.272676
1050.	7.0	10.5	0.7	-0.000000003	0.39851E 10	0.77456E 07	150.	15.0	0.222639
1600.	8.0	12.0	0.8	-0.000000002	0.56579E 10	0.73670E 07	200.	15.0	0.192811
2250.	9.0	13.5	0.9	-0.000000002	0.77651E 10	0.71011E 07	250.	15.0	0.172455
3000.	10.0	15.0	1.0	-0.000000001	0.10349E 11	0.68993E 07	300.	15.0	0.157429
3850.	11.0	16.5	1.1	-0.000000001	0.13453E 11	0.67383E 07	350.	15.0	0.145751

UNITS : R (IN.) a (IN.) DEFL. (IN.)
t (IN.) h (IN.) K_B(IN.-LB/RAD.)